Universes as Big-Data: Physics, Geometry & AI

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Quiver, Clusters, Moduli & Stability; Alastair King@60, Oxford, Jan 2023

Enriching the Maths/Physics Dialogue

- Alg./diff. Geometry/topology - Rep. Theo : the right language for physics
  - Gravity $\sim$ Ricci 2-form of Tangent bundles;
  - Elementary Particles $\sim$ irred reps of the Lorentz group and sections of bundles with Lie structure group; Interactions $\sim$ Tensor products of sections ...
  - **String theory: brain-child of gauge-gravity geometrization tradition**
- A new exciting era for synergy with (pure & computational) geometry, group theory, combinatorics, number theory: *Sage, M2, GAP, LMFDB, GrDB* are becoming indispensible tools for physicists
- Interdisciplinary enterprise: cross-fertilisation of particle/string theory, phenomenology, pure mathematics, computer algorithms, data-bases, ...

e.g., DICTIONARY: Quivers & QFT (Thank you, Alastair)

$$S = \int d^4x \, [\int d^2\theta d^2\bar{\theta} \, \Phi_i^\dagger e^V \Phi_i + \left(\frac{1}{4g^2} \int d^2\theta \ \mathrm{Tr}\, \mathcal{W}_\alpha \mathcal{W}^\alpha + \int d^2\theta \, W(\Phi) + \text{c.c.} \right)]$$

$$W = \text{superpotential} \qquad V(\phi_i, \bar{\phi}_i) = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{g^2}{4} (\sum_i q_i |\phi_i|^2)^2$$

- Encode into QUIVER (rep of finite labelled graph with relations):

k nodes, dim vec $(N_1, \ldots, N_k)$	$\prod_{j=1}^k U(N_j)$ gauge group
Arrow $i \to j$	bi-fund X_{ij} field $(\square, \overline{\square})$ of $U(N_i) \times U(N_j)$
Loop $i \to i$	adjoint ϕ_i field of $U(N_i)$
Cycles	Gauge Invariant Operator
2-cycles	Mass-terms
$W = \sum c_i$ cycles$_i$	Superpotenital
Relations	Jacobian of $W(\phi_i, X_{ij})$

- VACUUM $\sim V(\phi_i, \bar{\phi}_i) = 0 \Rightarrow$ $\begin{cases} \frac{\partial W}{\partial \phi_i, X_i} = 0 & \text{F-TERMS} \\ \sum_i q_i |\phi_i|^2 + q_k |X_k| = 0 & \text{D-TERMS} \end{cases}$

VMS: Vacuum Moduli Space

- $\mathcal{M}$:= **vacuum moduli space = space of sol'ns to F and D-flatness = affine algebraic variety** $\rightsquigarrow$ Representation (Quiver) Variety (GIT quotient)
- If $\mathcal{M}$ affine Calabi-Yau 3-fold: $\dim_{\mathbb{C}} \mathcal{M} = 3$ and locally $Ric(g^{\mu\nu}) = 0$
  - can realize in string theory $(10 = 4 + 2 \times 3)$ as D3-brane$\perp$ $\mathcal{M}$;
  - Dirichlet p-Branes: $p+1$ dimensional submanifold of $\mathbb{R}^{1,9}$ on which open-strings can end; D3-brane $\rightsquigarrow \mathbb{R}^{1,3}$
  - TRANSVERSE: local (affine, singular) Calabi-Yau 3-fold (cone over Sasaki-Einstein 5-manifold), crepant resolution to smooth CY3:

$$\begin{array}{ccc} & \text{Geom Engineer} & \\ \text{Affine CY3 } \mathcal{M} & \longleftrightarrow & \text{Quiver Gauge Theory} \\ & VMS & \end{array}$$

- Rmk: for N-branes, get $Sym^N \mathcal{M} = \mathcal{M}^N / \Sigma_N$

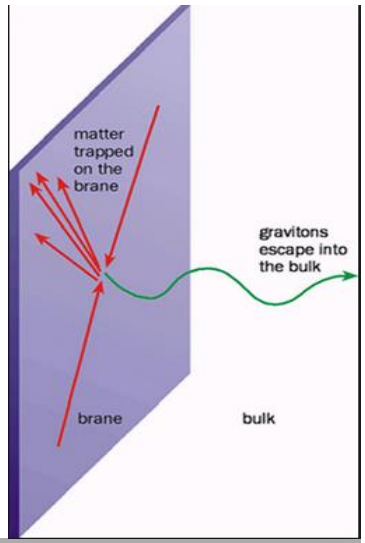
standard string paradigm: $10 = 4 + 3 \times 2$

Superstring Theory 9+1 d

Unified theory of quantum gravity

I. 6 Large Dim

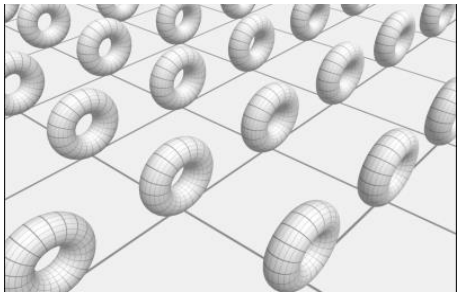
AdS/CFT
Brane World

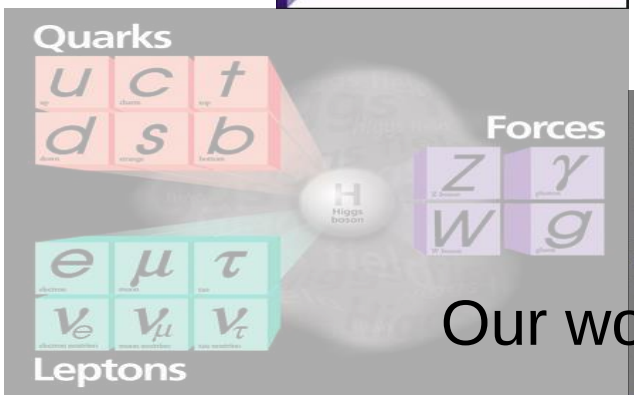


1. *Reduce Dim: 10 = 6+4*
2. *Break SUSY*

II. 6 small dim

Compactification





Our world 3+1d SU(3)xSU(2)xU(1) SM + GR

Toric CY3, Mirror Symmetry, Bipartite Tilings, Cluster Mutation, Seiberg Duality, Dessins, etc., etc.

- PHYSICISTS: Feng, Franco, Hanany, YHH, Kennaway, Martelli, Mekareeya, Seong, Sparks, Vafa, Vegh, Yamazaki, Zaffaroni ...
- THEOREM: [R. Böckland, N. Broomhead, A. Craw, A. King, K. Ueda ...] The (coherent component of) VMS as representation variety of a quiver is an affine (non-compact, possibly singular) toric Calabi-Yau variety of complex dimension 3 $\Leftrightarrow$ the quiver + superpotential is graph dual to a bipartite graph drawn on T^2, and cluster mutation of the quiver corresponds to Seiberg Duality.

Vacuum Degeneracy

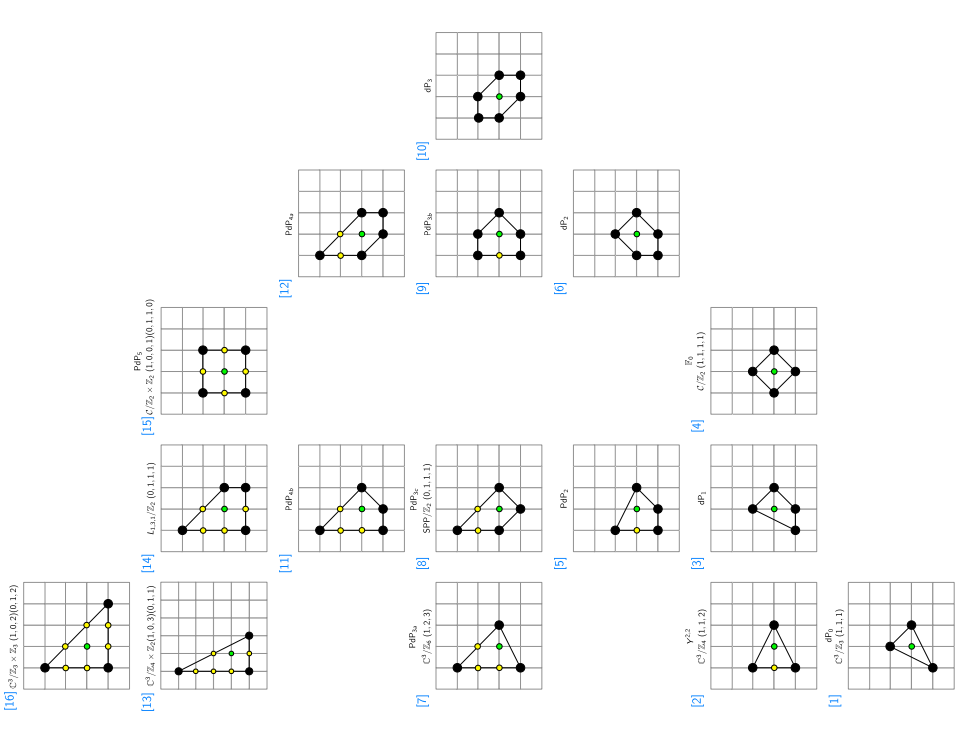
Perhaps the biggest theoretical challenge to string theory:

selection criterion??? metric on the landscape???

- Douglas (2003): Statistics of String vacua
- Kachru-Kallosh-Linde-Trivedi (2003): type II/CY estimates of 10^{500}
- Taylor-YN Wang (2015-7): F-theory estimates 10^{3000} to 10^{10^5}
- Basic Reason:

Algebraic Geometry $\rightsquigarrow$ Combinatorial Geometry $\rightsquigarrow$ Exponential Growth in dim

e.g., Borisov-Batyrev & Kreuzer-Skarke



- **Reflexive Polyhedra $\rightsquigarrow$ CY: anticanonical hypersurface in toric variety from Δ**
- **Dim 2: 16 up to $SL(2;\mathbb{Z})$ (Italian School 1890s)**
- **Dim 3: 4139 up to $SL(3;\mathbb{Z})$ (KS, 1999)**
- **Dim 4: 473800776 up to $SL(4;\mathbb{Z})$ (KS, 2000)**
- **Dim >4: Open ??**

GrDB: Brown, Kaspryzyk, Nil, Kahle, ...`http://www.grdb.co.uk/`

Altman-Gray-YHH-Jejjala-Nelson (2014): brute-force: $\sim 10^6$ up to $h^{1,1}=6$

Altman-Carifio-Halverson-Nelson (2018): estimated 10^{10^4} triangulations

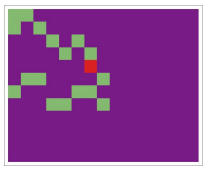
Demirtas-Long-McAllister-Stillman (2019): all triang $240 \leq h^{1,1} \leq 491$

Algebraic Geometry as Image Processing

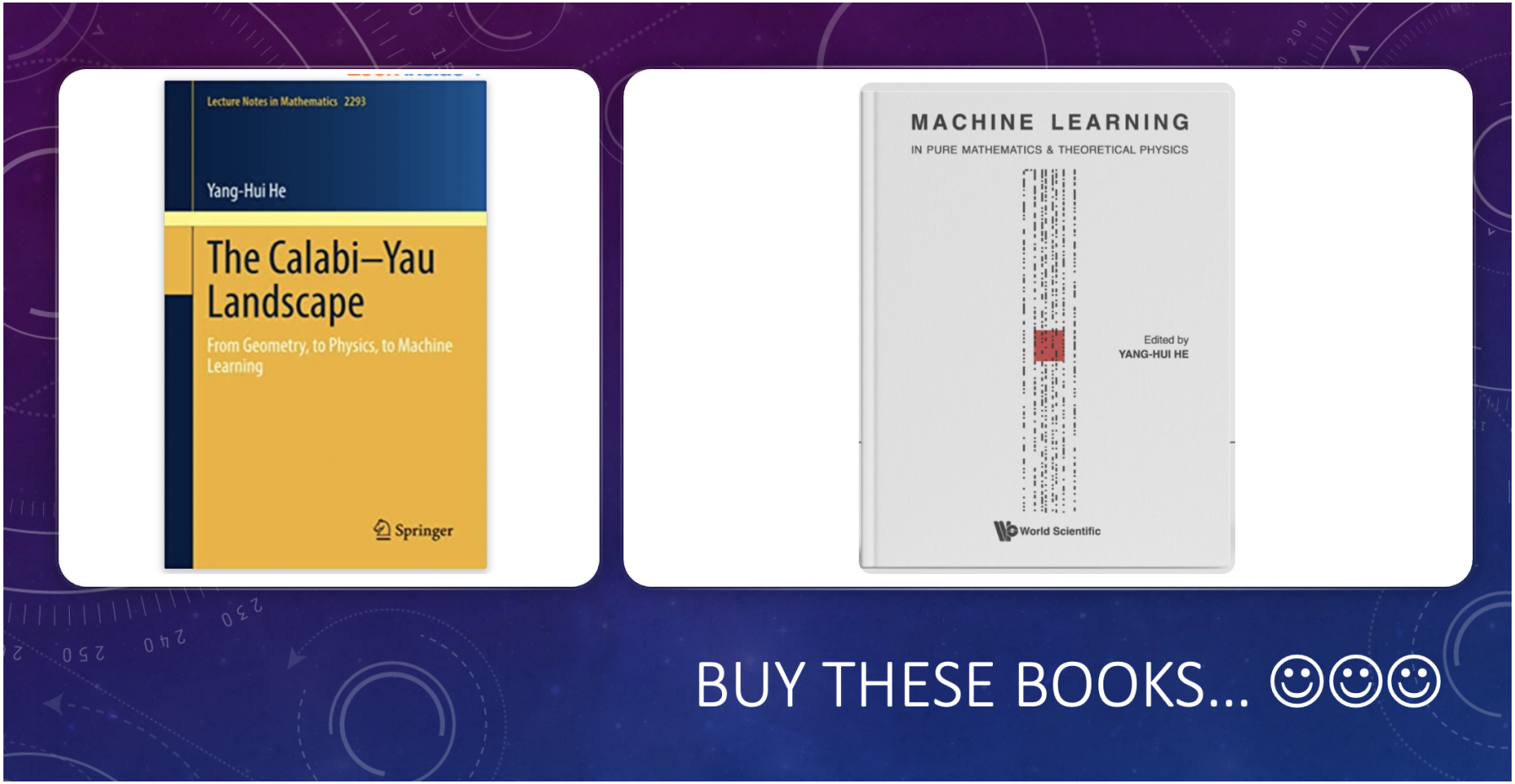
- A typical calculation:

$$X = \begin{pmatrix} 1&1&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0 \\ 0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&2&0&0 \\ 0&1&1&0&0&0&0&1 \\ 1&0&0&0&0&1&1&0 \\ 0&0&0&1&1&0&0&1 \end{pmatrix} \longrightarrow \boxed{\text{What Bourbaki teaches us}} \longrightarrow h^{2,1}(X) = 22$$

- Key to computational Algebraic Geometry: Gröbner basis, double-exponential complexity (unlike Gaussian elimination which is generalizes)
- [YHH 1706.02714] Deep-Learning the Landscape, *PLB 774, 2017*; (cf. Feature in *Science* , Aug, vol 365 issue 6452, 2019): think of it as an image processing problem



$\longrightarrow$ What Machine-Learning teaches us $\longrightarrow 22$



YHH: The CY Landscape: from Geometry, to Physics, to ML: Springer 2021;

YHH, Ed: ML in Pure Maths & Theoretical Physics: WS 2023

Machine Learning Mathematics

Why stop at string/geometry?

Review: YHH 2101.06317

Pattern Recognition: Human Eye

- $[0,0,1,0,0,1,0,0,1,0,0,1,\ldots]$

  multiple of 3 or not.

- $[1,1,1,0,1,1,0,1,1,0,1,0,0,1,1,0,0,1,0,1,\ldots]$

  Prime or Not for odd integers.

- $[1,0,0,1,0,1,0,0,1,1,0,0,0,1,1,1,0,0,0,0,\ldots]$

  Even/Odd of number of prime factors (Liouville Lambda)

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Pattern Recognition: Machine-Learning

- Binary Classification of a Binary Vector (sliding window of, say, length 100); supervised learning: predict next one, e.g., Prime/Not becomes:

$$\begin{array}{lll} \{0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,\ldots,0\} & \longrightarrow & 1 \\ \{1,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,\ldots,1\} & \longrightarrow & 0 \\ \{0,0,0,1,0,0,0,0,0,1,0,0,1,0,0,0,0,1,1,0,0,\ldots,0\} & \longrightarrow & 1 \\ \ldots & \ldots & \ldots \end{array}$$

- pass to standard classifiers: SVW, Bayes, Nearest Neighbour; NN of the form $\mathbb{R}^{100} \overset{\text{linear}}{\longrightarrow} \mathbb{R}^{20} \overset{\tanh}{\longrightarrow} \mathbb{R}^{20} \overset{\text{Round}\circ\sum}{\longrightarrow} \mathbb{R}$, your kitchen sink, ...
- take 50,000 samples, 20-80 cross-validation, record (precision, MCC)
- similar performance for most: Mod3: (1.0, 1.0); PrimeQ, after balancing: (0.8, 0.6); Liouville Λ: (0.5, 0.001)

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Thank you! Since 2017-

my fantastic students

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Combinatorics: **Johannes Hofscheier, Alexander Kasprzyk, Shiing-Tung Yau**

How does one *DO* mathematics, I ?

Russell-Whitehead *Principia Mathematica* [1910s] (Leibniz, Frege, ...) axiomatize maths, but . . . Gödel [1931] Incompleteness ; Church-Turing [1930s] Undecidability

Automated Theorem Proving (ATP) "The practicing mathematician hardly ever worries about Gödel"

- Newell-Simon-Shaw [1956] Logical Theory Machine: subset of *Principia*
- Type Theory [1970s] Martin-Löf, Coquand, . . . Coq: 4-color (2005); Feit-Thompson Thm (2012); Lean (2013); Univalent Foundation / Homotopy Type Theory [2006-] Voevodsky

Buzzard: "Future of Maths" 2019, ICM 2022

Davenport: ICM 2018 "Computer Assisted Proofs"

Szegedy: more extreme view, computers > humans @ chess (1990s); @ Go (2018); @ Proving theorems (2030)

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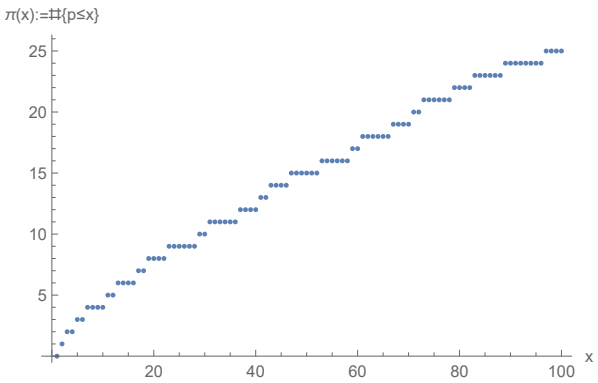
- Historically, Maths perhaps more Top-Down: practice before foundation
  - Countless examples: calculus before analysis; algebraic geometry before Bourbaki, permutation groups / Galois theory before abstract algebra . . .
  - A lot of mathematics starts with intuition, experience, and experimentation
- The best neural network of C18-19th? brain of Gauß ; e.g., age 16



(w/o computer and before complex analysis [50 years before Hadamard-de la Vallée-Poussin's proof]): PNT $\pi(x) \sim x/\log(x)$

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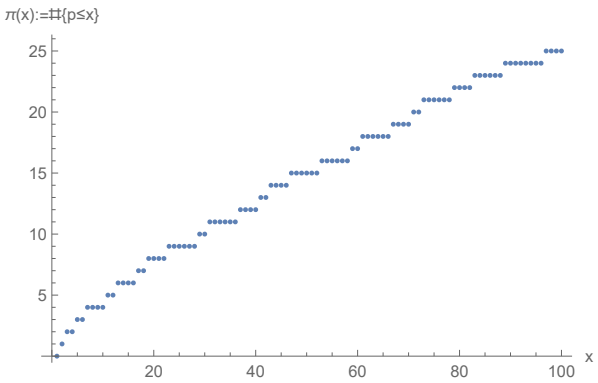

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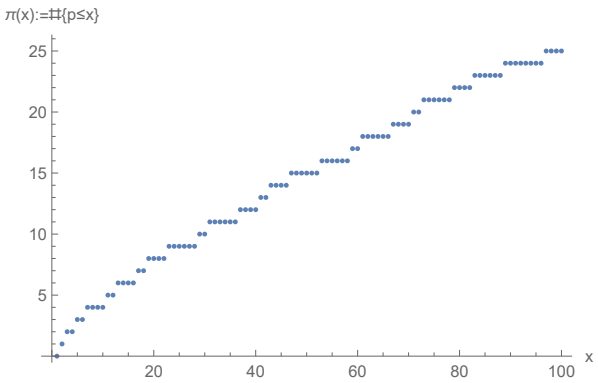

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Mathematical Data

- NOISELESS Data: different from real-world data to which ML is usually applied; If I gave you 100,000 cases of

$$\text{e.g. } \begin{pmatrix} 5&3&4&3&5&1&4&4&1&2\\ 5&0&4&5&3&4&4&2&2&4\\ 1&1&2&2&0&4&1&4&5&0\\ 5&0&1&1&0&2&0&5&0&1\\ 3&5&0&1&1&3&2&3&0&3\\ 3&2&2&3&0&0&2&2&1&0\\ 2&2&5&1&4&4&0&0&1&2\\ 5&0&0&0&4&5&0&4&1&1\\ 4&3&4&3&3&1&0&0&2&5\\ 2&0&5&0&3&0&4&4&1&5 \end{pmatrix}, \quad \text{or, labeled data e.g. } \begin{pmatrix} 5&3&4&3&5&1&4&4&1&2\\ 5&0&4&5&3&4&4&2&2&4\\ 1&1&2&2&0&4&1&4&5&0\\ 5&0&1&1&0&2&0&5&0&1\\ 3&5&0&1&1&3&2&3&0&3\\ 3&2&2&3&0&0&2&2&1&0\\ 2&2&5&1&4&4&0&0&1&2\\ 5&0&0&0&4&5&0&4&1&1\\ 4&3&4&3&3&1&0&0&2&5\\ 2&0&5&0&3&0&4&4&1&5 \end{pmatrix} \longrightarrow 3$$

- Q: Is there a pattern? Can one conjecture & then prove a formula?
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Example I: Representation/Group Theory

ML Algebraic Structures (GAP DB) [YHH-MH. Kim 1905.02263,]

- When is a Latin Square (Sudoku) the Cayley (multiplication) table of a finite group? Bypass quadrangle thm (0.95, 0.9)
- Can one look at the Cayley table and recognize a finite simple group?
  - bypass Sylow and Noether Thm; (0.97, 0.95) rmk: can do it via character-table T, but getting T not trivial
  - **SVM:** space of finite-groups (point-cloud of Cayley tables) seems to exist a hypersurface separating simple/non-simple

Example II: Combinatorics

[YHH-ST. Yau 2006.16619] Wolfram Finite simple graphs DB

- ML standard graph properties:

  ?acyclic (0.95, 0.96); ?planar (0.8, 0.6); ?genus $>, =, < 0$ (0.8, 0.7); ?$\exists$ Hamilton cycles (0.8, 0.6); ?$\exists$ Euler cycles (0.8, 0.6)

  (Rmk: NB. Only "solving" the likes of traveling salesman stochastically)

- spectral bounds ($R^2 \sim 0.9$) ...

- Recognition of Ricci-Flatness (0.9, 0.9) (todo: find new Ricci-flat graphs);

Example III: Quivers, Clusters, Brane setups, . . .

- [Bao-Franco-Hirst-Musiker, 2006.10783, Dechant-YHH-Heyes-Hirst 2203.13847] Recognition of mutation types (> 0.9)
- [Hirst-YHH-Peterken 2004.05218]: adjacency+permutation triple of dessin d'enfants; predicting transcendental degree > 0.9
- [Arias-Tamargo, YHH, Heyes, Hirst, Rodriguez-Gomez 2202.05845] Recognition of equivalence ($SL(2;\mathbb{Z})$, Seiberg, Hanany-Witten) of brane-webs
- [Cheung-Dechant-YHH-Heyes-Hirst-Li 2212.09771] learning Young tableaux representation of variables in Grassmannian cluster algebras (> 0.99)

Example IV: Number Theory

Arithmetic, A Classical Reprobate? (prime numbers are Difficult!)

- [YHH 1706.02714, 1812.02893:]
  - Predicting primes $2 \to 3,\ 2,3 \to 5,\ 2,3,5 \to 7$; no way
  - PrimeQ: (0.7, 0.8); Sarnak's Challenger of Liouville Lambda (0.5, 0.001)
- [Alessandretti-Baronchelli-YHH 1911.02008]

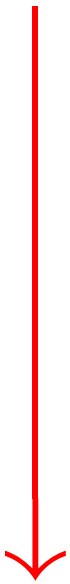
  ML/TDA@Birch-Swinnerton-Dyer $Ш$ and Ω ok with regression & decision trees: RMS < 0.1; Weierstrass $\to$ rank: random
- Arithmetic Geometry: A Modern Hope? YHH-KH Lee-Oliver
  - 2010.01213: Complex Multiplication, Sato-Tate $(0.99 \sim 1.0, 0.99 \sim 1.0)$
  - 2011.08958: Number Fields: rank and Galois group (0.97, 0.9)
  - 2012.04084: BSD from Euler coeffs, integer points, torsion (0.99, 0.9); Tate-Shafarevich $Ш$ (0.6, 0.8) [Hardest quantity of BSD]

Clearly useful for maths and physics

- Conjecture Formulation e.g.,
  - '19 YHH-Kim: separating hyperplane - simple/non-simple groups; open
  - '19 Brodie-Constantin-Lukas: exact formulae for cohomo surf.; proved.
  - '20 YHH-Lee-Oliver: L-coefs and integer pt./torsion on ell; proved.
  - '20 Craven-Jejjala-Par: Jones poly best-fit function; open
  - '22 DeepMind Collab bounds on volume conjecture for knots
  - ...
- Speed up & Improve Accuracies e.g.,
  - computing/estimating (top.inv., charges, etc) MUCH FASTER
  - '19 Ashmore-YHH-Ovrut: speed up Donaldson alg@CY metric 10-100
  - '20 Douglas et al., Anderson et al. accuracy improvement on Donaldson 10-100 times
  - ...

An Inherent Hierarchy?

- ML the structure of mathematics: YHH 2101.06317
- In decreasing precision/increasing difficulty:



numerical

string theory $\to$ algebraic geometry over $\mathbb{C} \sim$ arithmetic geometry

algebra

string theory $\to$ combinatorics

analytic number theory

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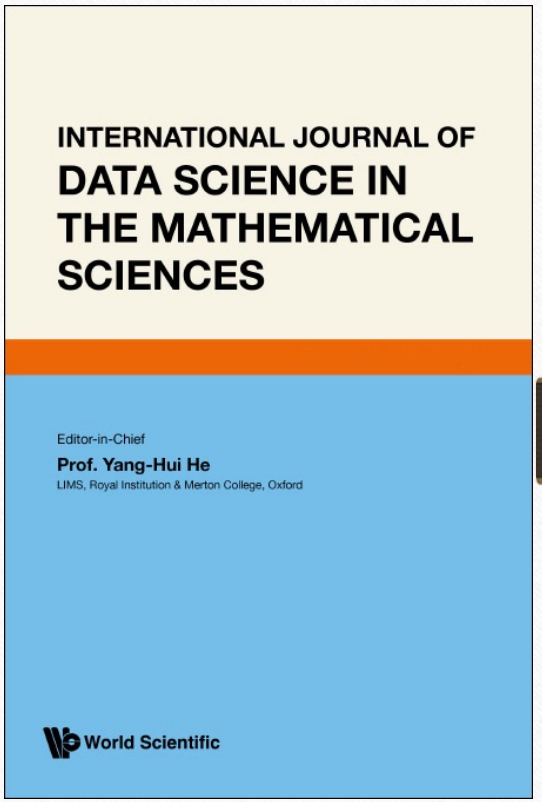

World Scientific
Connecting Great Minds

The London Institute for Mathematical Sciences

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- Housed in the Faraday Suites of the Royal Institution of Great Britain
- 23 Themes: `https://lims.ac.uk/`
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Arnold Felowships

Landau Fellowships

Thank You

Happy 60th

Birthday Alastair!!

The Proper Way $\mathcal{O}(e^{e^d})$

- Recall Hodge decomposition $H^{p,q}(X) \simeq H^q(X, \wedge^p T^\star X) \rightsquigarrow$

$$H^{1,1}(X) = H^1(X, T_X^\star), \qquad H^{2,1}(X) \simeq H^{1,2} = H^2(X, T_X^\star) \simeq H^1(X, T_X)$$

- Euler Sequence for subvariety $X \subset A$ is short exact:

$$0 \to T_X \to T_M|_X \to N_X \to 0$$

- Induces long exact sequence in cohomology:

$$\begin{array}{ccccccc} 0 & \to & \cancel{H^0(X, T_X)}^{\,0} & \to & H^0(X, T_A|_X) & \to & H^0(X, N_X) \to \\ & \to & \boxed{H^1(X, T_X)} & \stackrel{d}{\to} & H^1(X, T_A|_X) & \to & H^1(X, N_X) \to \\ & \to & H^2(X, T_X) & \to & \cdots & & \end{array}$$

- Need to compute Rk(d), cohomology and $H^i(X, T_A|_X)$ (Cf. Hübsch)

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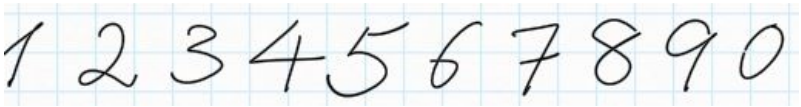
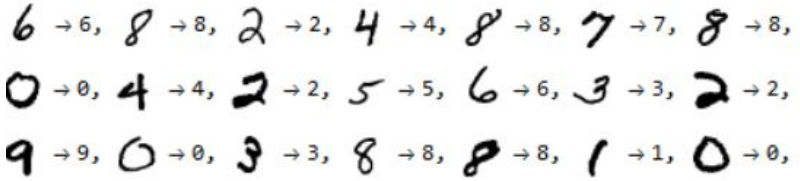
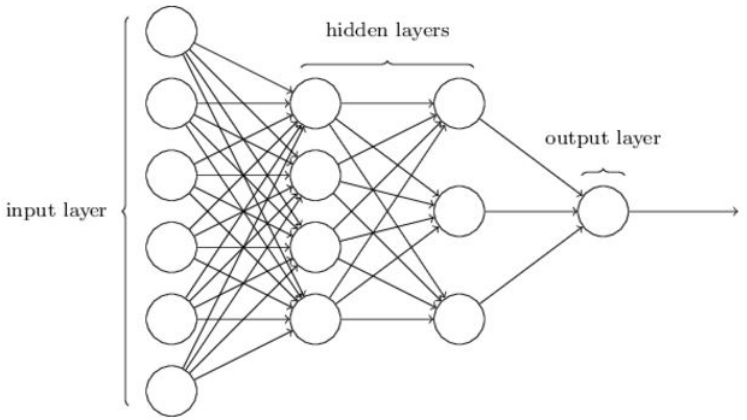
The Neural Network Approach

- Bijection from  to $\{1, 2, \ldots, 9, 0\}$?
- Take large sample, take a few hundred thousand (e.g. NIST database)





- Data = Training Data ⊔ Validation Data

  Test trained NN on validations data to see accuracy performance

Universal Approximation Theorems

Large Depth Thm: (Cybenko-Hornik) For every continuous function $f : \mathbb{R}^d \to \mathbb{R}^D$, every compact subset $K \subset \mathbb{R}^d$, and every $\epsilon > 0$, there exists a continuous function $f_\epsilon : \mathbb{R}^d \to \mathbb{R}^D$ such that $f_\epsilon = W_2(\sigma(W_1))$, where σ is a fixed continuous function, $W_{1,2}$ affine transformations and composition appropriately defined, so that $\sup_{x \in K} |f(x) - f_\epsilon(x)| < \epsilon$.

Large Width Thm: (Kidger-Lyons) Consider a feed-forward NN with n input neurons, m output neuron and an arbitrary number of hidden layers each with $n + m + 2$ neurons, such that every hidden neuron has activation function φ and every output neuron has activation function the identity. Then, given any vector-valued function f from a compact subset $K \subset \mathbb{R}^m$, and any $\epsilon > 0$, one can find an F, a NN of the above type, so that $|F(x) - f(x)| < \epsilon$ for all $x \in K$.

ReLU Thm: (Hanin) For any Lebesgue-integral function $f : \mathbb{R}^n \to \mathbb{R}$ and any $\epsilon > 0$, there exists a fully connected ReLU NN F with width of all layers less than $n + 4$ such that $\int_{\mathbb{R}^n} |f(x) - F(x)| dx < \epsilon$.

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