Errata and updates for "Renormalization"

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The publisher's webpage for John Collins's book "Renormalization: An introduction to renormalization, the renormalization group and the operator product expansion QCD" (Cambridge University Press, 1986, ISBN = 9781009401807) can be accessed from `https://doi.org/10.1017/9781009401807` The book is available as an Open Access pdf file (since July 2023), as well as in print form (hardback and paperback). The author's webpage for the book is `https://www.cantab.net/users/johncollins/ren-book/` and includes this list of errata.

The latest published version is dated July 2023, but with content corresponding to the paperback reprint of 1987.

1 Corrections

- This list has correction relative to the 1987 reprint, which is the current version. For extra corrections if your book is older than that, see Sec. 2.
- Eq. (2.11.5): Missing) after first $\omega(x)$.
- P. 30, Eq. (2.12.4): In the exponent, there should be "$\int d^4x$" before "$\mathcal{L}_{\rm gc}$", so that the equation reads
$$\Delta = \int [dc_\alpha][d\bar{c}_\alpha] \exp\left(i \int d^4x \mathcal{L}_{\rm gc} \right). \tag{2.12.4}$$
- Eq. (2.12.9): delete 'i', so that the equation reads
$$\mathcal{L}_{\rm gf} = -F_\alpha^2/(2\xi), \tag{2.12.9}$$
- Eq. (2.13.6), first line: c_β should be $\overline{c_\beta}$.
- P. 44, second paragraph of Sec. 3.2: It is stated that "all the graphs in Fig. 3.2.1 are one-particle-reducible". However, it is common in calculations to omit external line factors, so it might appear that graph (c) is 1PI, contrary to the statement. But

for the purposes of this discussion, we retain all external lines, and graph (c) is then one-particle-reducible, as is seen by cutting any of the external lines.

What is important in this context is that the only UV divergence in graph (c) is in the self-energy subgraph, as will become clearer in later sections. The issue in this section is to show in simple cases what happens given that the counterterm to the self-energy subgraph is implemented by a term in the interaction Lagrangian. As explained in the following paragraphs, this implementation of renormalization enables each one-loop self-energy graph to be replaced by its renormalized value.

The paragraph should be replaced by

> One property should be clear. This is that the divergences come from subgraphs all of whose lines are part of a loop. A general way of characterizing these subgraphs is to define the concept of a one-particle-irreducible graph or subgraph. A one-particle-irreducible (1PI) graph is one which is connected and cannot be made disconnected by cutting a single line. A graph which is not 1PI is called one-particle-reducible (1PR).
>
> All of the graphs in Fig. 3.2.1 are 1PR, since they all have one or more lines that when cut leave the graph in two disconnected pieces. *Note that graph (c) is intended to include its external lines. Cutting any of them splits the graph into disconnected components.* However, it is often convenient to amputate the external lines, i.e., to remove the corresponding propagator factors. Then graph (c) becomes a 1PI graph with two nested loops. In contrast, the other graphs remain 1PR even after amputation of their external lines. The self-energy subgraph of Fig. 3.1.1 consisting of the two lines in the loop is similarly 1PI. This identical subgraph occurs several times in the graphs of Fig. 3.2.1. Moreover the only UV divergences in the graphs are in the self-energy subgraphs, as will become clearer for graph (c) in later sections.

- P. 70, at the beginning of the second line below the equation containing $f(p^2) = (p^2 + A)/(p^2 + B)$, replace '$l = 1$' by '$l = 0$'.

- P. 74: In the second line of the second paragraph after Eq. (4.3.3), the second "covariant" should be "contravariant", so that the paragraph reads:

> A contravariant tensor may be defined by specifying its components. But a covariant tensor ω is fundamentally a linear function acting on contravariant tensors: $\omega(T)$. We can write $\omega(T) = \omega_{ij}T^{ij}$ only if the sum converges.

- Eq. (4.5.7): On the *left*-hand side of the second line, the subscript '${}_{(\omega)}$' should be replaced by '$_{(\omega+1)}$'. Then the second line of the equation should read:

$$\gamma^{2\omega+1}_{(\omega+1)} = \begin{pmatrix} 0 & \mathrm{i}\hat{\gamma}_{(\omega)} \\ \mathrm{i}\hat{\gamma}_{(\omega)} & 0 \end{pmatrix} \tag{4.5.7}$$

- P. 99, Eq. (5.2.16): The subscript '$l < k$' should be replaced by '$l > k$'.
- P. 102, just after Eq. (5.3.2), the last sentence of the paragraph should be emphasized and expanded: "*In a Feynman graph for a Green's function there is a well-known external momentum at the vertices for the external fields. But at an interaction vertex, we can think of there also being an external momentum which in the usual situation is zero: $p_i = 0$. In normal usage, one does not typically treat interaction vertices as having external momenta. But for the purposes of the discussion at this point, it is convenient to change the convention. Then all vertices are treated on the same footing.*"
- P. 107: After Eq. (5.4.3), add a footnote:

  > Observe that the isolated 'G' at the beginning of the right-hand-side of this equation is the same as '$U(G)$' in earlier equations. There we distinguished the concepts, on the one hand of a graph G, with its structure of lines and vertices, and on the other hand the corresponding integral and value '$U(G)$'. But to de-clutter the notation, we now overload the symbol 'G' to mean both the graph and its integral.

- P. 114, Eq. (5.6.10): 'G_n' should be replaced by 'G_N'.
- At the end of the line after Eq. (14.1.8), insert "Here, the symbol j^{had} for the hadronic (i.e., QCD) part of the electromagnetic current has been replaced by j, which is the standard notation in this context."
- Eq. (14.1.9): Replace $|0\rangle$ by $|p\rangle$.
- Eq. (14.1.10): Insert "$/M^2$" at the end of the first line.
- Eq. (14.2.2): Delete extra) at the end of equation.
- Eq. (14.4.7): Replace $|0\rangle$ on the left-hand side by $|p\rangle$.

2 Corrections made in 1987 reprint (current version)

- P. iv: Add dedication for book "To Mary".
- P. 82, Eq. (4.4.8): The factor "$\exp(-B^2/A)$" on the *first* line should be replaced by "$\exp(Ak^2 + 2B \cdot k)$". (The factor "$\exp(-B^2/A)$" on the *second* line is unchanged.)

- P. 172, Eq. (7.1.12): There should be a factor 1/2 with the mass term on the first line. I.e., insert "/2" at the end of the first line.

- P. 181, Eq. (7.3.4b): The first factor of m^2 should be replaced by m^{-2}.

- P. 182, Eq. (7.3.11): In the *second* line, in the formula for g_0, insert a factor g in front of $c_1(g)$ in the numerator, so that the numerator is $a_1(g) - \frac{3}{2} g c_1(g)$. In contrast, no such correction is needed on the fourth line.

- P. 183, Eq. (7.3.12a) should read

$$\begin{aligned}
\beta(g, d) &= \left(\frac{d}{2} - 3\right) \frac{\left[g + \dfrac{a_1(g) - \frac{3}{2} g c_1(g)}{6-d} + \text{higher poles}\right]}{\left[1 + \dfrac{a_1'(g) - \frac{3}{2} g c_1'(g) - \frac{3}{2} c_1(g)}{6-d} + \text{higher poles}\right]} \\
&= (d/2 - 3) g + \frac{1}{2}\left(1 - g\frac{\partial}{\partial g}\right)\left[\frac{3}{2} g c_1(g) - a_1(g)\right] + \text{poles (that cancel)} \\
&= (d/2 - 3) g + \bar{\beta}(g), \qquad (7.3.12a)
\end{aligned}$$

- P. 183, Eq. (7.3.12b): In the first line, in the formula for γ_m, insert a factor g after the factor $(d/2 - 3)$. Thus the square-bracket factor is the same as the formula in the immediately preceding line.

- P. 183, Eq. (7.3.14): Insert μ in front of $\dfrac{\mathrm{d}}{\mathrm{d}\mu}$, so that the equation reads

$$\gamma = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln Z; \qquad (7.3.14)$$

- P. 184, Eq. (7.3.15): Change the second two "d"s to partial derivatives, ∂. I.e., all the derivatives on the *right*-hand side of the equation are partial derivatives, in contrast to the left-hand side.

- P. 186, Eqs. (7.4.3) and (7.4.4): In the powers of ζ, replace $N/2$ by N.

- P. 207, Eq. (7.10.1): The factor "$Z^{-N\ 2}$" should read "$Z^{-N/2}$".

- And see the other corrections in Sec. 1. These weren't made in the 1987 reprint.