Errata and updates for "Foundations of Perturbative QCD"

John Collins

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The publisher's webpage for John Collins's book "Foundations of Perturbative QCD" (Cambridge University Press, 2011, ISBN 9781009401845) is accessed from `https://doi.org/10.1017/9781009401845` The book is available as an Open Access pdf file (since July 2023) as well as in print form (hardback and paperback). The author's webpage for the book is `https://www.cantab.net/users/johncollins/qcdbook/` and includes this list of errata.

The latest published version is dated July 2023, but with content corresponding to the paperback edition of November 2013.

Note: There are three sections in this document, for different classes of change.

1 Corrections relative to published edition of May 2011, that have been corrected in the 2013 printing

In addition to the corrections listed here from the original edition to the November 2013 printing, there are some other changes (as opposed to corrections of errors) given in Sec. 2, and there are further updates and corrections relative to the November 2013 printing, given in Sec. 3.

- Eq. (5.47): $\lambda_S^2 k^2$ should be replaced by $\lambda_S^2 \overline{k}^2$.
- Eq. (7.51): $k_n, \ldots k_1$ should have superscript $+$: k_n^+, k_n^+, k_{n-1}^+ in line 2, and k_n^+, k_1^+ in line 3.
- Eq. (9.24) for the gluon-in-gluon DGLAP kernel should have the 1/3 with the delta-function replaced by 1/6 so that the equation reads:

$$\frac{g^2}{16\pi^2} P_{gg}^{[1]}(z) = \frac{g^2}{8\pi^2} \left\{ 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(z-1) \frac{11C_A - 4n_f T_R}{6} \right\}, \quad (9.24)$$

- Caption of Fig. 10.7 (p. 332): At the end of the caption, $\ln(Q^2/m^2)$ should be replaced by $\ln(Q/m)$.
- P. 333, end of second line before Eq. (10.35): change $\ln k_{\rm T}^2$ to $\ln k_{\rm T}$.
- P. 333, 3 lines after heading "Original graph": Change "falls of" to "falls off".
- P. 351, 5 lines after (10.71): Change "place" to "plane".
- P. 410, second line of (11.18): Change "$\partial \cdot A\alpha$" to "$\partial \cdot A^\alpha$".
- P. 410, second line of (11.18): Delete first minus sign, and change second minus sign to plus sign.
- P. 427, in heading of Sec. 12.1, and in the second line of the following text, the dashes in "structure-function" should be ordinary hyphens.
- Fig. 12.11 (p. 452): In the upper left label of a quark line, the label should be $k_{1,1}$, the same as in Fig. 12.12.
- P. 464, Eq. (12.80): Right-hand side needs a factor of C_F to be inserted.
- P. 513. End of section heading for 13.12 should read $q_{h\ T}$.
- Eq. (13.110) (p. 530): g_{j/H_A} in the first exponent should be changed to g_{f/H_A}.
- Eq. (13.116) (p. 532): The sign of the exponent in the Fourier transform should be reversed, so that the exponential reads $e^{-i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}$.
- Eq. (A.16) (p. 587): The "2" in the denominator (multiplying the square root) should be replaced by "4", so that the equation reads

$$d\sigma = (2\pi)^4 \delta^{(4)}\Big(p_1 + p_2 - \sum_j q_j\Big) \prod_{j=1}^{n} \frac{d^3\boldsymbol{q}_j}{(2\pi)^3 2E_{\boldsymbol{q}_j}} \times \\ \times \frac{\big|\mathcal{M}(\boldsymbol{p}_1, \boldsymbol{p}_2; \boldsymbol{q}_1, \dots, \boldsymbol{q}_n)\big|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}. \tag{A.16}$$

- In addition, all the corrections in later sections should be made.

2 Updates relative to published edition of May 2011, that have been made in the 2013 printing

This section lists changes (as opposed to corrections of errors) made in the November 2013 printing relative to the original edition. For errors corrected in the November 2013 printing, see Sec. 1. For further updates and corrections relative to the November 2013 printing see Sec. 3.

- Bibliography: Vossen et al. (2009) should be replaced by Vossen et al. (2011). (SPIRES code Vossen:2011fk:

  A. Vossen, R. Seidl, I. Adachi, H. Aihara, T. Aushev, V. Balagura, W. Bartel, M. Bischofberger *et al.*, "Observation of the interference fragmentation function for charged pion pairs in e^+e^- annihilation near $\sqrt{s} = 10.58$ GeV," Phys. Rev. Lett. 107, 072004 [arXiv:1104.2425].

- Index entry for "Boer-Mulders function" should have additional page reference to p. 569. (This is in middle of Sec. 14.5.4.)

- Add the following entries to the index

  DGLAP equation, calculations of kernel, 286–293

  DGLAP equation, results for kernel, 288, 293, 464

- In addition, all the corrections in later sections should be made.

3 Corrections and updates relative to published paperback edition of November 2013

In the paperback edition of November 2013, all the corrections and updates have been made that were listed in Secs. 1 and 2. In addition, the following further correction(s) are needed (they apply to both editions):

- P. 11, second and third lines from bottom: Replace "being much less than" by "being not much less". (Interestingly, the file supplied to the publisher had the correct statement.)

- Two lines above Eq. (2.27), change "lepton-parton scattering" to "virtual-photon-parton process".

- Eq. (3.86) should read

$$\alpha_{s,N} = \alpha_{s,N+1} \left[1 - \frac{\alpha_s}{3\pi} T_F \ln \frac{\mu^2}{m_{N+1}^2} + O(\alpha_s^2) \right]. \tag{3.86}$$

  That is, the "2π" in the coefficient inside the brackets should be replaced by "3π".

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- Eq. (4.38) should read

$$\frac{R(Q^2)}{6\pi} = 2\Im\Pi(Q^2 + i0) = \frac{\Pi(Q^2 + i0) - \Pi(Q^2 - i0)}{i}. \tag{4.38}$$

- In the line after Eq. (5.9), "$\nu = p \cdot q$" should be replaced by "$\nu = P \cdot q$" (with an upper case P).

- On p. 96, at the beginning of the first full paragraph, replace "Just in" by "Just as in".

- On p. 115, 7 lines after Eq. (5.35), the word "there" is doubled, but should not be.

- On p. 117, the beginning of the second sentence of the second paragraph of Sec. 5.5.5 ("Thus the power of Q for the whole graph …") should be modified to read as follows: "Since we defined the power of Q associated with a graph to be the power *relative* to the tree graph, it follows that the power for the whole graph …".

- P. 125: in the last paragraph of the page, k^+ and k^- should be exchanged.

- P. 159: To exercise 5.3 should be added the following as a footnote: "I have succeeded in constructing a suitable proof in Collins (2020)."

- In Eq. (6.28), a slash is needed between $\partial(1-x)^n$ and ∂x, as in $\partial(1-x)^n/\partial x$.

- In Eq. (6.76), change "target" to "target helicity" (twice).

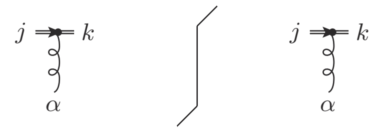
- In Eq. (6.78), change "target" to "target transverse spin" (twice).

- In Eq. (6.110), insert Tr before each occurrence of $\frac{\gamma^+}{2}$, just as in the second line of (6.89).

- In the paragraph containing Eq. (6.113), the momentum variable p should be upper case: P.

- On p. 217, in the second line of the second paragraph of Sec. 7.2, remove a repeated "to".

- In Fig. 7.10, there should be a reversal of the sign of the Feynman rule for the gluon-to-Wilson-line coupling to the *right* of the final-state cut in the gluon pdf. That is, Fig. 7.10 should be modified to:



Quark pdf, bare fields:	$-ig_0 n^\mu (t_\alpha)_{kj}$	$ig_0 n^\mu (t_\alpha)_{kj}$
Quark pdf, renorm. fields:	$-ig_0 Z_3^{1/2} n^\mu (t_\alpha)_{kj}$	$ig_0 Z_3^{1/2} n^\mu (t_\alpha)_{kj}$
Gluon pdf, bare fields:	$g_0 n^\mu f_{k\alpha j}$	$-g_0 n^\mu f_{k\alpha j}$
Gluon pdf, renorm. fields:	$g_0 Z_3^{1/2} n^\mu f_{k\alpha j}$	$-g_0 Z_3^{1/2} n^\mu f_{k\alpha j}$

Fig. 7.10. Feynman rules for vertex on Wilson lines in parton densities. Here, $n^\mu = \delta^\mu_- = (0, 1, \mathbf{0}_\mathrm{T})$. In the Wilson line for a gluon pdf, the generating matrix for the adjoint representation was used: $(T_\alpha)_{kj} = i f_{k\alpha j}$. The sign of the vertex is reversed compared with Collins and Soper (1982b), and corresponds to the sign of the coupling in our Lagrangian, whose Feynman rules are in Fig. 3.1

- On p. 251, two lines before the start of Sec. 8.3, "neglect k_T with respect to l_T" should be changed to "neglect l_T with respect to k_T".

- Eq. (9.41) should be corrected to

$$\hat{F}_{2g} = \sum_j \frac{g^2 e_j^2 T_F}{4\pi^2} \left(\frac{16\pi\mu^2 z}{Q^2(1-z)} \right)^\epsilon \frac{z}{\Gamma(1-\epsilon)} \int_{-1}^{1} \mathrm{d}\cos\theta \, (\sin\theta)^{-2\epsilon} \times$$
$$\times \left\{ \frac{1}{\sin^2\theta} \left[1 - \frac{2z(1-z)}{1-\epsilon} \right] - \frac{1}{2(1-\epsilon)} + \frac{3-2\epsilon}{(1-\epsilon)^2} z(1-z) \right\}$$
$$- \text{ term from graph (c)}, \tag{9.41}$$

where relative to the published edition, the term $\frac{-2+5\epsilon}{4(1-\epsilon)^2}$ is replaced by $-\frac{1}{2(1-\epsilon)}$. Since at $\epsilon = 0$ the value of the term is unchanged, the correction has no effect on the coefficient function in (9.43).

- Eq. (9.48a) should be corrected to

$$\hat{F}_{2j}(\mathrm{a}) = \frac{g^2 e_j^2 C_F}{8\pi^2} \left(\frac{16\pi\mu^2}{\hat{s}} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int_{-1}^{1} \mathrm{d}\cos\theta \, (\sin\theta)^{-2\epsilon} \times$$
$$\times \left\{ \frac{z(1-z)(1-\epsilon)}{1+\cos\theta} + \frac{1}{2}(3-2\epsilon) z^2 (1-\cos\theta) \right\}. \tag{9.48a}$$

Here, the 4 multiplying $(3 - 2\epsilon)$ in the published version was corrected to 1/2.

- In Eq. (9.48b) "(b)" should be replaced by "(b+h.c.)".

- Eq. (9.48c) should be corrected to

$$\hat{F}_{2j}(\mathrm{c}) = \frac{g^2 e_j^2 C_F}{8\pi^2} \left(\frac{16\pi\mu^2}{\hat{s}}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \int_{-1}^{1} \mathrm{d}\cos\theta \; (\sin\theta)^{-2\epsilon} \times \\ \times \frac{z(1-\epsilon)}{4(1-z)}(1+\cos\theta). \tag{9.48c}$$

  Here, an overall factor 1/4 was inserted.

- Eq. (9.52) should be corrected to

$$(\mathrm{e}+\mathrm{h.c.}) : \frac{S_\epsilon}{2\epsilon}\delta(z-1) = \frac{1}{2\epsilon}\delta(z-1) + \delta(z-1)\,\frac{1}{2}[\ln(4\pi) - \gamma_{\mathrm{E}}] + O(\epsilon). \tag{9.52}$$

  Here, a factor 1/2 was inserted in the $[\ln(4\pi) - \gamma_{\mathrm{E}}]$ term, and a missing ")" was inserted.

- Eq. (9.53) should be corrected to

$$(\mathrm{f–h}) : \frac{S_\epsilon}{\epsilon}\left[\frac{z(1+z^2)}{(1-z)_+} + 2\delta(z-1)\right]. \tag{9.53}$$

  Here, the 5/2 multiplying the delta function was replaced by 2.

- Eq. (9.54) should be corrected to

$$\hat{F}_{2j}(Q^2, z; \alpha_s, \mu) = e_j^2\delta(z-1) + \\ + \frac{g^2 e_j^2 C_F}{16\pi^2} z \Bigg\{ 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 3\left(\frac{1}{1-z}\right)_+ - 2(1+z)\ln(1-z) - \\ - 2\frac{1+z^2}{1-z}\ln z + 6 + 4z - \left(\frac{2\pi^2}{3} + 9\right)\delta(1-z)\Bigg] \\ + \left[2(1+z^2)\frac{1}{(1-z)_+} + 3\delta(z-1)\right]\ln\frac{Q^2}{\mu^2}\Bigg\} \\ + O(g^4). \tag{9.54}$$

  Here, the $\ln\frac{Q^2}{\mu^2}$ term was added.

- P. 326, last line: Change $(k_A + k_{AS}^2)$ to $(k_A + k_{AS})^2$.

- Eq. (10.24): Change P_{HA} to P_{HB}.

- P. 334: The inline equation two lines above Eq. (10.37) should read $\frac{1}{2}(y_{p_A} - y_{p_B})\ln(Q/m) = \frac{1}{4}\ln^2(Q^2/m^2)$.

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- P. 367: In Fig. 10.17, the lowest S in (a) and the lowest S' in (b) should be A and A' respectively, i.e., the figure is:

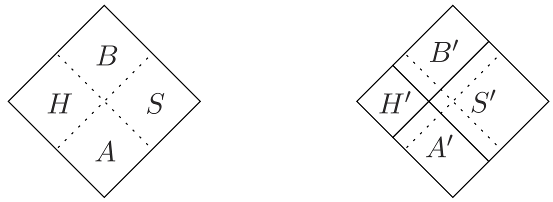

Fig. 10.17. (a) Partition of graph for Sudakov form factor by subgraphs for a region R. (b) Partition for a smaller region $R' < R$. The dotted lines indicate the boundaries of the subgraphs for the first region.

- P. 373, 5 lines above start of Sec. 10.8.8: Change $y_{u_2} \to \infty$ to $y_{u_2} \to -\infty$.
- P. 381, item 4 in list: Change "light-like" to "non-light-like".
- P. 383, second line: Change "before the removing the UV regulator" to "before removing the UV regulator".
- P. 402: In Eqs. (11.4)–(11.6), subscript Lorentz indices should be replace by superscripts, and vice versa, so that the paragraph reads:

  Of the lines entering the hard scattering H from the collinear subgraph C, let N be gluons, for which we write the polarization sum as

$$H \cdot C = H^{\mu_1 \dots \mu_N} \prod_{j=1}^{N} g_{\mu_j \nu_j} \, C^{\nu_1 \dots \nu_N}. \tag{11.4}$$

  Let k_j be the momentum of gluon j flowing into H. The largest term in its polarization sum has $\mu_j = -$, $\nu_j = +$, and we manipulate it into a form suitable for the use of Ward identities. Accordingly, we make a Grammer-Yennie decomposition

$$g_{\mu_j \nu_j} = K_{\mu_j \nu_j} + G_{\mu_j \nu_j}, \tag{11.5}$$

  where

$$K_{\mu_j \nu_j} = \frac{k_{j\,\mu_j} w_{2\,\nu_j}}{k_j \cdot w_2 - i0}, \quad \text{and} \quad G_{\mu_j \nu_j} = g_{\mu_j \nu_j} - \frac{k_{j\,\mu_j} w_{2\,\nu_j}}{k_j \cdot w_2 - i0}, \tag{11.6}$$

  and the vector w_2 projects onto plus components of momentum: $w_2 = (0, 1, \mathbf{0}_{\mathrm{T}})$. Then from (11.4), we get a sum of terms which we label by saying that each of the gluons is a K gluon or a G gluon according to which term in (11.5) is used.

- P. 407, second line of third paragraph: Replace "the all the" by "all the".
- P. 416: In Fig. 11.6(a), the arrow on the right-hand fermion should be reversed.
- P. 428, Eq. (12.8): In the first line, the prefactor $\sqrt{1-\frac{4m^2}{Q^2x^2}}$ should be removed, so that the equation reads

$$E\frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}} = \frac{2\alpha^2}{Q^4}\left[F_1(x,Q) + \frac{x}{4}\left(1-\frac{4m^2}{Q^2x^2}\right)\sin^2\theta\, F_2(x,Q)\right] \tag{12.8}$$
$$\simeq \frac{2\alpha^2}{Q^4}\left[F_1(x,Q) + \frac{x}{4}\sin^2\theta\, F_2(x,Q)\right]. \tag{1}$$

- P. 435: In the first line of Eq. (12.35), $N_{j/h}$ should be replaced by $N_{h/j}$.
- P. 436: In the exponent of the first line of Eq. (12.39), $\boldsymbol{k}_\mathrm{T}$ should be replaced by $\boldsymbol{k}_{h\,\mathrm{T}}$.
- P. 446, 8 lines below Eq. (12.49b), change "k^+, but not k^-, is not to be deformed" to "k^+, but not k^-, is to be deformed".
- P. 450: The two bulleted items starting "For a quark" and "For an *anti*quark" should be replaced by

  - For a quark exiting the hard scattering to collinear subgraph α, we project its Dirac spinor onto a massless on-shell wave-function multiplying H by inserting a factor $\tilde{\not{w}}_\alpha \not{w}_\alpha/(2\tilde{w}_\alpha\cdot w_\alpha)$. Relative to the light-front coordinates defined by w_α and $\tilde{w}_\alpha$, this matrix is $\gamma^+\gamma^-/2$; in this case w_α and $\tilde{w}_\alpha$ are in the + and − directions respectively, and the situation corresponds to item (e) on p. 329.
  - For an *anti*quark exiting the hard scattering, we use the projector $\not{w}_\alpha\tilde{\not{w}}_\alpha/(2w_\alpha\cdot\tilde{w}_\alpha)$.

- P. 455: Replace Eq. (12.61) by

$$W^{\mu\nu}([f],p) = \sum_{N\geq 2}\sum_{j_1}\cdots\sum_{j_N}\prod_{\alpha=1}^{N}\left[\int\frac{\mathrm{d}^nk_\alpha}{(2\pi)^n}\right]\prod_{\alpha=1}^{N}\left[C_{(\alpha),j_\alpha}(k_\alpha)\right]\int\frac{\mathrm{d}^nk_S}{(2\pi)^n}S_{j_1,\dots j_N}(k_S)\times$$
$$\times\, H_{j_1,\dots j_N}(\hat{k}_1,\dots,\hat{k}_N)\, f(\hat{k}_1+\dots+\hat{k}_N)\prod_{\alpha=2}^{N}\left(\frac{|\hat{\boldsymbol{k}}_\alpha|}{|\boldsymbol{k}_\alpha|}\right)^{n-2}. \tag{13.61}$$

This has a rearrangement of summation signs and integrals to try to make the meaning clearer. In addition, the subscript j_α on H was replaced by j_N.

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- P. 456: Eq. (12.65) should be replaced by:

$$W^{\mu\nu}(q,p) = \int \mathrm{d}\hat{k}_1^+ \sum_{j_1} \left[\int \frac{\mathrm{d}k_1^- \, \mathrm{d}^{n-2}\boldsymbol{k}_{1\,\mathrm{T}}}{(2\pi)^n} C_{(1),j_1}(k_1,p) \right] \times$$
$$\times \sum_{N\geq 2} \sum_{j_2} \cdots \sum_{j_N} \prod_{\alpha=2}^{N} \left[\int \frac{\mathrm{d}^{n-1}\hat{\boldsymbol{k}}_\alpha}{2|\hat{\boldsymbol{k}}_\alpha|(2\pi)^{n-1}} \right] \prod_{\alpha=2}^{N} \left[2\hat{k}_\alpha^+ \int \frac{\mathrm{d}k_\alpha^-}{2\pi} C_{(\alpha),j_\alpha}(k_\alpha) \right] \times$$
$$\times \int \frac{\mathrm{d}^n k_S}{(2\pi)^4} S_{j_1,\dots j_N}(k_S)(2\pi)^n \delta^{(n)}\big(q - \sum_{\alpha=1}^{N} \hat{k}_\alpha\big) H_{j_1,\dots j_N}(\hat{k}_1,\dots,\hat{k}_N). \quad (13.65)$$

  This has a rearrangement of summation signs and integrals to try to make the meaning clearer. In addition, the subscript j_α on S and H was replaced by j_N, and a sum over N was inserted.

- P. 459: In Eq. (12.71), the directions of the arrows on the Wilson lines should be reversed.

- P. 460: Eq. (12.72) should be replaced by:

$$d_{(0)\,h/g}(z) = \frac{z^{n-3}}{N_{c,\,\mathrm{gluon}}(n-2)k^+} \sum_X \int \frac{\mathrm{d}x^-}{2\pi} e^{ik^+x^-} \times \quad (12.72)$$
$$\times (-g_{\lambda\lambda'}) \langle 0| \, G^{+\lambda}_{(0),\,b}(x/2) \, \left[W_A(\infty, x^-/2; \tilde{w}_1)^\dagger \right]_{bc} \, |p, X, \mathrm{out}\rangle \times$$
$$\times \langle p, X, \mathrm{out}| \, \left[W_A(\infty, -x^-/2; \tilde{w}_1) \right]_{cd} \, G^{+\lambda'}_{(0),\,d}(-x/2) \, |0\rangle \,, \quad (2)$$

  This has certain signs with x and x^- changed.

- P. 485: In the third line of Sec. 13.3.1, "have only have" should be replaced by "have only".

- P. 485: In the first line of the last paragraph, "for a soft momenta" should be replaced by "for soft momenta".

- After (13.22), add a statement that the trace is over color as well as Dirac indices.

- P. 490: In the fourth line of item 5, "only valid only" should be replaced by "valid only".

- P. 492. In Eq. (13.28) p_{hT} should be changed to p_T.

- P. 495: In the third and fourth lines of (13.34) insert a factor of N_c.

- P. 504: In Eq. (13.52), the factor $(4\pi\mu^2)^\epsilon$ should be replaced by $(4\pi^2\mu^2)^\epsilon$, i.e., the π is squared.

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- P. 521: In the the delta function at the end of first line of Eq. (13.90), $(k-p^2)$ should be replaced by $(k-p)^2$.
- P. 525: In the last sentence of the last line, "be" needs to be inserted so that the sentence reads "But there can be practical difficulties with low-order estimates."
- In the two lines before Eq. (13.115), change the in-line equation for q to $q+\sum_\alpha \hat{k}_{A,\alpha} = \sum_\beta \hat{k}_{B,\beta}$.
- In Eq. (13.115), remove the minus sign in the + component of k, so that the equation reads

$$\hat{k}_{A,\gamma} = \left(xP^+_{A,\gamma}, 0, \mathbf{0}_\mathrm{T}\right), \qquad \hat{k}_{B,\gamma} = \left(0, \frac{Q^2}{2xP^+_{A,\gamma}}, \mathbf{0}_\mathrm{T}\right). \tag{13.115}$$

- P. 574, line 9: replace "an full" by "a full".
- Add the following to the bibliography: "Collins, J. (2020). A new and complete proof of the Landau condition for pinch singularities of Feynman graphs and other integrals. arXiv:2007.04085."
- Add the following items to the index: