Course Syllabus: Numerical Approximation Theory (数值逼近论)

Offered by	School of Mathematics
Course code	MA5107
Semester	Spring 2019
Target students	Graduate level students in mathematics
Pre-requisites	Mathematical Analysis, Linear Algebra, and Differential Equations at undergraduate level. Knowledge of programming languages such as MATLAB would be useful.
Assessment	80% examination, 20% homework
Class hours	Thursdays 5-8 (14:20 to 18:00)

Course outline and schedule

Broadly speaking, Numerical Approximation Theory is the study of algorithms with the use of numerical approximations, in many disciplines of mathematics such as linear algebra and differential equations. The subject naturally has numerous applications in fields such as engineering, natural science, social science, business science, and so on. This course will consist of three chapters, with roughly the following schedule.

Weeks 1 to 6. *Numerical linear algebra.* Various types of matrix factorisations including Schur, singular value decomposition, QR, LU, and Cholesky. Matrix transformations including Gaussian elimination and Householder. Various algorithms and their flop counts. Eigenvalue problems and their algorithms including the power, inverse, and Rayleigh quotient iterations.

Weeks 7 to 11. *Interpolation theory.* Interpolation using fundamental, Newton and Hermite polynomials. Difference formulas and divided differences. Symmetric functions. Legendre, Jacobi, and Chebyshev polynomials. Minimax approximation. Bernstein polynomials and Weierstrass approximation theorem. Splines and B-splines.

Weeks 12 to 16. *Numerical methods for differential equations.* Approximations to solutions of differential equations by methods of Euler, Runge-Kutta, Adams-Bashforth, Adams-Moulton, and various others. Collocation methods. Butcher tableaus. Stiff differential equations and A-stability. Two-point boundary value problems. Shooting methods. Elliptic, parabolic, and hyperbolic PDEs, stencil methods, and their stability.

Weeks 17 to 18. Revision; and additional (non-examinable) material.

Week 19. Final examination.

Textbooks

The course will not be based on any particular textbook. There are many good relevant books, including following.

1. Kendall E. Atkinson, Weimin Han, David E. Stewart, "Numerical Solution of Ordinary Differential Equations", John Wiley & Sons, Inc., 2009.
2. J.C. Butcher, "Numerical Methods for Ordinary Differential Equations" (3rd ed), John Wiley & Sons, Inc., 2016.
3. Gwynne A. Evans, Jonathan M. Blackledge, Peter D. Yardley, "Numerical Methods for Partial Differential Equations", Springer-Verlag, 2000.
4. Zhilin Li, Zhonghua Qiao, Tao Tang, "Numerical Solutions of Differential Equations", Cambridge University Press, 2018.
5. George M. Phillips, "Interpolation and Approximation by Polynomials", Springer-Verlag, 2003.
6. Lloyd N. Trefethen, David Bau III, "Numerical Linear Algebra", SIAM, 1997.

Evaluation and grading policy

Homework assignments will be given out throughout the semester. There will be a final **closed book** examination in week 19. The homeworks and the final examination will contribute 20% and 80% respectively towards the final mark. There will be no mid-term examination.

Pass mark: **At least 70% for the final mark.**

Instructor and contact details

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