Four Lectures on Proof-theoretic Semantics

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Schedule

1. What is Proof-theoretic Semantics (P-tS)?
   - Inferentialism.
   - Consequence.
   - Proof-theoretic Validity (P-tV).
2. Base-extension Semantics (B-eS):
   - B-eS for Intuitionistic Propositional Logic.
   - Naturality, categorically speaking.
   - B-eS and P-tV.

Schedule

3. Reductive logic, tactical proof, and logic programming:
   - Reductive Logic and P-tV.
   - Tactical Proof.
   - Remarks on Logic Programming and Coalgebra.
4. Modal and Substructural Logics, Resource Semantics, and Modelling:
   - B-eS for Modal Logics.
   - B-eS for Substructural Logics.
   - Resource Semantics and Modelling with B-eS.

Schedule

- Most of what we will introduce will be quite new to most people, with a fairly significant philosophical basis, and with quite a lot of ground to be covered.
- Our approach will mainly be conceptual, with little detailed, formal proof.
- Nevertheless, the formal details of everything we cover are available in books and papers that will be referenced.

Lecture 4: Modal and Substructural Logics, Resource Semantics, and Modelling

- First, two key logical foundations:
  - Modal and epistemic logics: there is a great deal of current work on P-tS for modal logics (mostly B-eS). We won't get into it today, but do please ask if you're interested. There are a couple of recent papers in the *Logic J. of the IGPL*.
  - Substructural logics: today's topics will look briefly at IMLL and at BI. There is also a strong line of work that is addressing full linear logic — beyond our scope here — led by YII Buzoku and Elaine Pimentel.
- Second, systems concepts:
  - Resource semantics: from BI and Separation Logic
  - Distributed systems: The 'distributed systems metaphor' provides a convenient conceptual language.

What is a 'System'?

With a little abstraction, we can employ the 'distributed systems metaphor':

- a collection of interconnected *locations*,
- at which are situated *resources*,
- relative to which *processes* execute — consuming, creating, moving, combining, and otherwise manipulating resources as they evolve, so delivering the system's services.
- many examples, including buildings, businesses, computers, communication networks (e.g., the internet), and so on — think about them in terms of their architecture and the services that they deliver.

Example: Vending Machine

- *locations*: customer, vending machine
- *resources*: money (i.e., kr in Iceland), chocolate bars
- *processes* (@C): 200kr is consumed, 1 chocolate bar is produced

Figure: Reykjavík University

- Later, we'll explore a more substantial example in some detail.

Resource Semantics

How can we reason about such systems?

Resource Semantics

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Definition (Resource Semantics)

A *resource semantics* for a system of logic is

- an interpretation of its formulae as assertions about states of processes, and
- expressed in terms of the resources manipulated by those processes.

For more on this, see: David Pym. Resource semantics: logic as a modelling technology. ACM SIGLOG News, April 2019, Vol. 6, No. 2, 5–41. And references therein.

Resource Semantics

This definition requires a few notes:

- we intend no restriction on the assertions — e.g., permit 'higher-order' assertions about state transitions.
- we intend to express all kinds of processes relevant to the domain
- we require accounting for counting, composition, comparison, sharing, and separation of resources

Intuitionistic Multiplicative Linear Logic (IMLL)

$$\frac{}{\phi \vdash \phi}\; Ax$$

$$\frac{\Gamma \vdash \phi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \phi \otimes \psi}\; \otimes I \qquad \frac{\Gamma \vdash \phi \otimes \psi \quad \Delta, \phi, \psi \vdash \chi}{\Gamma, \Delta \vdash \chi}\; \otimes E$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \multimap \psi}\; \multimap I \qquad \frac{\Gamma \vdash \phi \multimap \psi \quad \Delta \vdash \phi}{\Gamma, \Delta \vdash \psi}\; \multimap E$$

Resource Semantics and IMLL

Propositions are read directly as resources:

- $(200kr \multimap KitKat)$ and $(240kr \multimap KAFFE)$
- $440kr \multimap (KitKat * KAFFE)$
- and so on.

In the presence of the additives, & and $\oplus$, things are a bit more interesting.

Base-extension Semantics for IMLL

- We require this semantics to be context-sensitive — this is to ensure that we can capture the multiplicativity that is inherent in IMLL's inference rules.
- Therefore, we enrich support $\Vdash$ with a multiset of atoms T — 'atomic resources'.
- The details are in:

Alexander Gheorghiu, Tao Gu, and David Pym.
Proof-theoretic Semantics for Intuitionistic Multiplicative Linear Logic. Studia Logica, 2024.
https://doi.org/10.1007/s11225-024-10158-6.

Base-extension Semantics for IMLL

Here are some clauses:

$$
\begin{array}{llr}
\Vdash^{S}_{\mathcal{B}} \phi \otimes \psi & \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T \, \forall p \, (\phi \fatcomma \psi \Vdash^{T}_{\mathcal{C}} p \implies \Vdash^{S \fatcomma T}_{\mathcal{C}} p) & (\otimes) \\
\Vdash^{S}_{\mathcal{B}} \phi \multimap \psi & \text{iff} \quad \phi \Vdash^{S}_{\mathcal{B}} \psi & (\multimap) \\
\Gamma \Vdash^{S}_{\mathcal{B}} \phi & \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T \, (\Vdash^{T}_{\mathcal{C}} \Gamma \implies \Vdash^{S \fatcomma T}_{\mathcal{C}} \phi) & (\text{Inf}) \\
\Vdash^{S}_{\mathcal{B}} \Gamma_1 \fatcomma \Gamma_2 & \text{iff} \quad \exists T_1, T_2 (S = T_1 \fatcomma T_2, \Vdash^{T_1}_{\mathcal{B}} \Gamma_1 \text{ and } \Vdash^{T_2}_{\mathcal{B}} \Gamma_2) & (\fatcomma)
\end{array}
$$

This is all quite intuitive — e.g., $(\otimes)$ recalls $\otimes E$,

$$
\dfrac{\phi \otimes \psi \qquad \begin{array}{c} [\phi, \psi] \\ \vdots \\ p \end{array}}{p} \; \otimes E
$$

Resource Semantics and BI

- A alternative resource semantics is associated with BI, the logic of bunched implications.
- In BI, the 'resource interpretation' resides in its semantics — as represented in Kripke-style models.
- Let's first quickly review BI.
- BI can be seen as the direct (essentially free) combination of IMLL and IPL, with minimal conditions required for the combination. 'Bunching' is the key one.

Bunched Implications, BI

- Contexts are no longer finite sequences.
- Instead, finite trees:
  - internal vertices labelled with either ‘ , ’ (comma, multiplicative) or ‘ ; ’ (semicolon, additive)
  - leaves labelled with formulae
- $\Gamma ::= \emptyset_m \mid \emptyset_a \mid \Gamma \, , \, \Gamma \mid \Gamma \, ; \, \Gamma$
- Substitution of subtrees.

Bunched Implications, BI

$$\frac{}{\phi \vdash \phi}\ Ax \qquad \frac{\Gamma(\Delta) \vdash \phi}{\Gamma(\Delta') \vdash \phi}\ (\Delta \equiv \Delta')\quad E$$

$$\frac{\Gamma(\Delta\,;\Delta) \vdash \phi}{\Gamma(\Delta) \vdash \phi}\ C \qquad \frac{\Gamma(\Delta) \vdash \phi}{\Gamma(\Delta\,;\Delta') \vdash \phi}\ W$$

$$\frac{\Gamma \vdash \phi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \phi * \psi}\ {*I} \qquad \frac{\Gamma \vdash \phi * \psi \quad \Delta(\phi\,,\psi) \vdash \chi}{\Delta(\Gamma) \vdash \chi}\ {*E}$$

$$\frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi -\!\!* \psi}\ {-\!\!*\ I} \qquad \frac{\Gamma \vdash \phi -\!\!* \psi \quad \Delta \vdash \phi}{\Gamma, \Delta \vdash \psi}\ {-\!\!*\ E}$$

$$\frac{\Gamma \vdash \phi \quad \Delta \vdash \psi}{\Gamma; \Delta \vdash \phi \wedge \psi}\ \wedge I \qquad \frac{\Gamma \vdash \phi \wedge \psi \quad \Delta(\phi\,;\psi) \vdash \chi}{\Delta(\Gamma) \vdash \chi}\ \wedge E$$

$$\frac{\Gamma; \phi \vdash \psi}{\Gamma \vdash \phi \supset \psi}\ \supset I \qquad \frac{\Gamma \vdash \phi \supset \psi \quad \Delta \vdash \phi}{\Gamma; \Delta \vdash \psi}\ \supset E$$

Can also have disjunction, but not needed today.

Resource Semantics and BI

The simple semantics, given in Lecture 3, will do for now — (possibly partial) ordered monoid, $(R, \circ, e, \sqsubseteq)$ ... . See Gheorghiu and Pym, Semantical Analysis of the Logic of Bunched Implications, *Studia Logica* 2023, for the full story.

$$
\begin{array}{rcl}
r \models p & \text{iff} & r \in \mathcal{V}(p) \\
r \models \bot & \text{never} & \\
r \models \top & \text{always} & \\
r \models \phi \vee \psi & \text{iff} & r \models \phi \text{ or } r \models \psi \\
r \models \phi \wedge \psi & \text{iff} & r \models \phi \text{ and } r \models \psi \\
r \models \phi \to \psi & \text{iff} & \text{for all } s \sqsubseteq r,\ s \models \phi \text{ implies } s \models \psi
\end{array}
$$

$$
\begin{array}{rcl}
r \models I & \text{iff} & r \sqsubseteq e \\
r \models \phi * \psi & \text{iff} & \text{there are worlds } s \text{ and } t \text{ such that} \\
 & & r \sqsubseteq (s \circ t) \text{ and } s \models \phi \text{ and } t \models \psi \\
r \models \phi \mathrel{-\!\!*} \psi & \text{iff} & \text{for all } s \text{ such that } (r \circ s) \text{ and } s \models \phi, \\
 & & r \circ s \models \psi
\end{array}
$$

Resource Semantics and BI

The resource reading of BI is very different form that of IMLL :

- The reading resides in models, with structure of composition (combination) and comparison.
- It is based on *sharing* and *separation*.
- The resources required to support $\phi * \psi$ are the composition of those required for ϕ and those required for ψ.
- If the resource required for $\phi \multimap \psi$ be combined with that required for ϕ, then the result is the resource required for ψ.
- Major example: Separation Logic.

Base-extension Semantics for BI

- Generalize the treatment of IMLL.
- We have *primitive* additive and multiplicative conjunctions and implications — this is useful for modelling.
- Collections of formulae are now 'bunches' — e.g., $a \, , (b \, ; c)$.
- Enrich support $\Vdash$ with bunches of atoms S, 'atomic resources'.
- We need a notion of a 'contextual bunch', intimately bound up with substitution into bunches. We won't get into the technical details here.
- This is a collection of (atomic) bunches characterized as a 'bunch with a hole' — $S(\cdot)$ amounts to the collection of bunches of shape S instantiated with T; that is, $S(T)$.

Base-extension Semantics for IMLL

Here are some clauses:

$$\Vdash_{\mathcal{B}}^{S} \phi \otimes \psi \quad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T \, \forall p \, (\phi \fatcomma \psi \Vdash_{\mathcal{C}}^{T} p \implies \Vdash_{\mathcal{C}}^{S \fatcomma T} p) \qquad (\otimes)$$

Note:

$$\frac{\phi \otimes \psi \qquad \begin{matrix}[\phi, \psi] \\ p\end{matrix}}{p}$$

$$\Vdash_{\mathcal{B}}^{S} \phi \multimap \psi \quad \text{iff} \quad \phi \Vdash_{\mathcal{B}}^{S} \psi \qquad (\multimap)$$

$$\Gamma \Vdash_{\mathcal{B}}^{S} \phi \quad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T \, (\Vdash_{\mathcal{C}}^{T} \Gamma \implies \Vdash_{\mathcal{C}}^{S \fatcomma T} \phi) \qquad (\text{Inf})$$

$$\Vdash_{\mathcal{B}}^{S} \Gamma_1 \fatcomma \Gamma_2 \quad \text{iff} \quad \exists T_1, T_2 (S = T_1 \fatcomma T_2, \Vdash_{\mathcal{B}}^{T_1} \Gamma_1 \text{ and } \Vdash_{\mathcal{B}}^{T_2} \Gamma_2) \qquad (\fatcomma)$$

Base-extension Semantics for BI

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$$\Vdash^{S}_{\mathcal{B}} \phi * \psi \quad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T(\cdot) \, \forall p \, (\phi \mathbin{,} \psi \Vdash^{T(\cdot)}_{\mathcal{C}} p \implies \Vdash^{T(S)}_{\mathcal{C}} p) \qquad (*)$$

$$\Vdash^{S}_{\mathcal{B}} \phi \mathrel{-\!\!*} \psi \quad \text{iff} \quad \phi \Vdash^{S \mathbin{,} (\cdot)}_{\mathcal{B}} \psi \qquad (\mathrel{-\!\!*})$$

$$\Gamma \Vdash^{S(\cdot)}_{\mathcal{B}} \phi \quad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T \, (\Vdash^{T}_{\mathcal{C}} \Gamma \implies \Vdash^{S(T)}_{\mathcal{C}} \phi) \qquad (\text{Inf})$$

As with IMLL, note that $(*)$ tracks the elimination rules.

Base-extension Semantics for BI

And some more clauses:

$$\Vdash^{S}_{\mathcal{B}} \phi \wedge \psi \quad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B} \, \forall T(\cdot) \, \forall p \, (\phi \fatsemi \psi \Vdash^{T(\cdot)}_{\mathcal{C}} p \implies \Vdash^{T(S)}_{\mathcal{C}} p) \qquad (\wedge)$$

$$\Vdash^{S}_{\mathcal{B}} \phi \supset \psi \quad \text{iff} \quad \phi \Vdash^{S \fatsemi (\cdot)}_{\mathcal{B}} \psi \qquad (\supset)$$

Note here that $(\wedge)$ tracks the generalized elimination rule:

$$\frac{\phi \wedge \psi \qquad \begin{array}{c}[\phi \, ; \psi] \\ p\end{array}}{p}$$

Details of BI's B-eS in: Alexander Gheorghiu, Tao Gu, and David Pym. Proof-theoretic-semantics for the Logic of Bunched Implications. Submitted, 2024. https://arxiv.org/abs/2311.16719.

Modelling with Proof-theoretic Semantics I

In general, for the base-extension semantics for some logic — e.g., IPL, ILL, BI:

$$\Gamma \Vdash_{\mathcal{B}}^{S(\cdot)} \phi \qquad \text{iff} \quad \forall \mathcal{C} \supseteq \mathcal{B}, \forall U \in \mathbb{R}(\mathbb{A}), \quad \text{if } \Vdash_{\mathcal{C}}^{U} \Gamma, \text{then } \Vdash_{\mathcal{C}}^{S(U)} \phi \tag{Gen-Inf}$$

This admits the kind of resource semantics we desire:

- ϕ is an assertion describing (a possible state of) the system
- Γ specifies a policy describing the executions of a system's processes
- $S(\cdot)$ is some 'contextual' collection of atomic resources
- $\mathcal{B}, \mathcal{C}$ are models of the systems — that is, $\Vdash_{\mathcal{C}}^{U} \Gamma$ says that $\mathcal{C}$ is a model of policy Γ when supplied with resource U.

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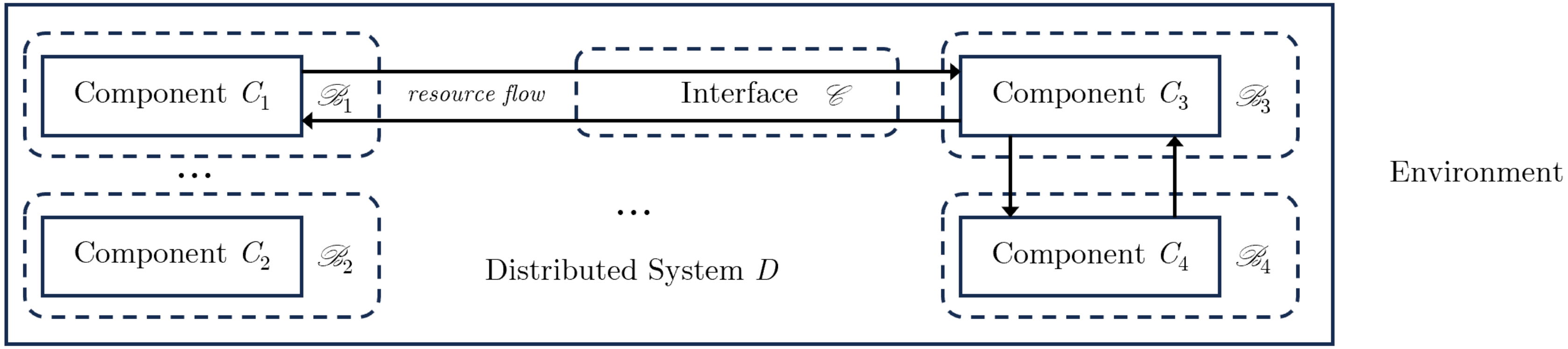
If policy Γ were to be executed with contextual resource $S(\cdot)$ based on the model $\mathcal{B}$, then the result state would satisfy ϕ.

Modelling with Proof-theoretic Semantics II

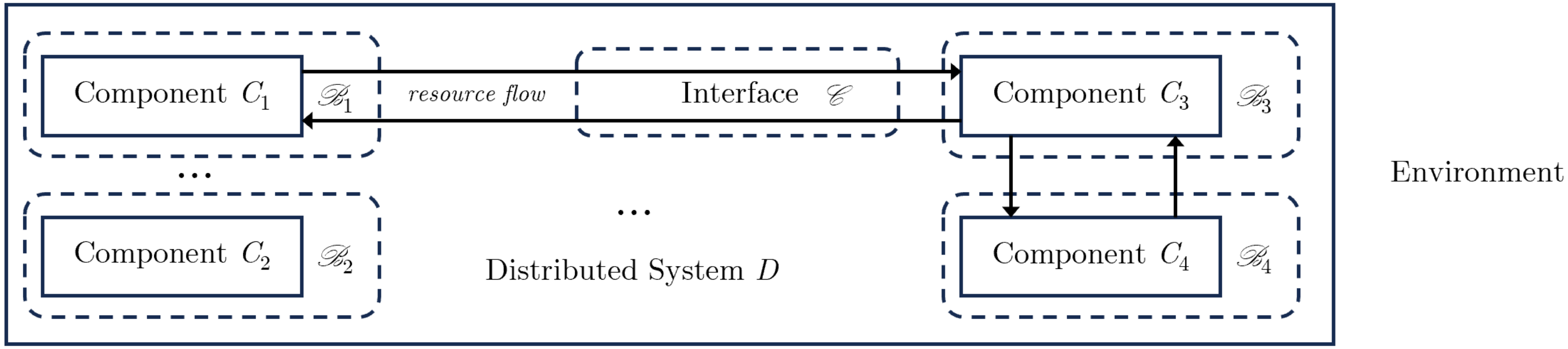
- Recall that we employ the 'distributed systems metaphor'.
- See, for example,
  - David Pym. Resource semantics: logic as a modelling technology. ACM SIGLOG News, April 2019, 6(2), 5–41
  - T. Caulfield, M.-C. Ilau, and D. Pym. Engineering Ecosystem Models: Semantics and Pragmatics. In *Proc. 13th SIMUtools 2021*. Springer, 2021
  - links at my page

and (many) references therein.

Modelling with Proof-theoretic Semantics II



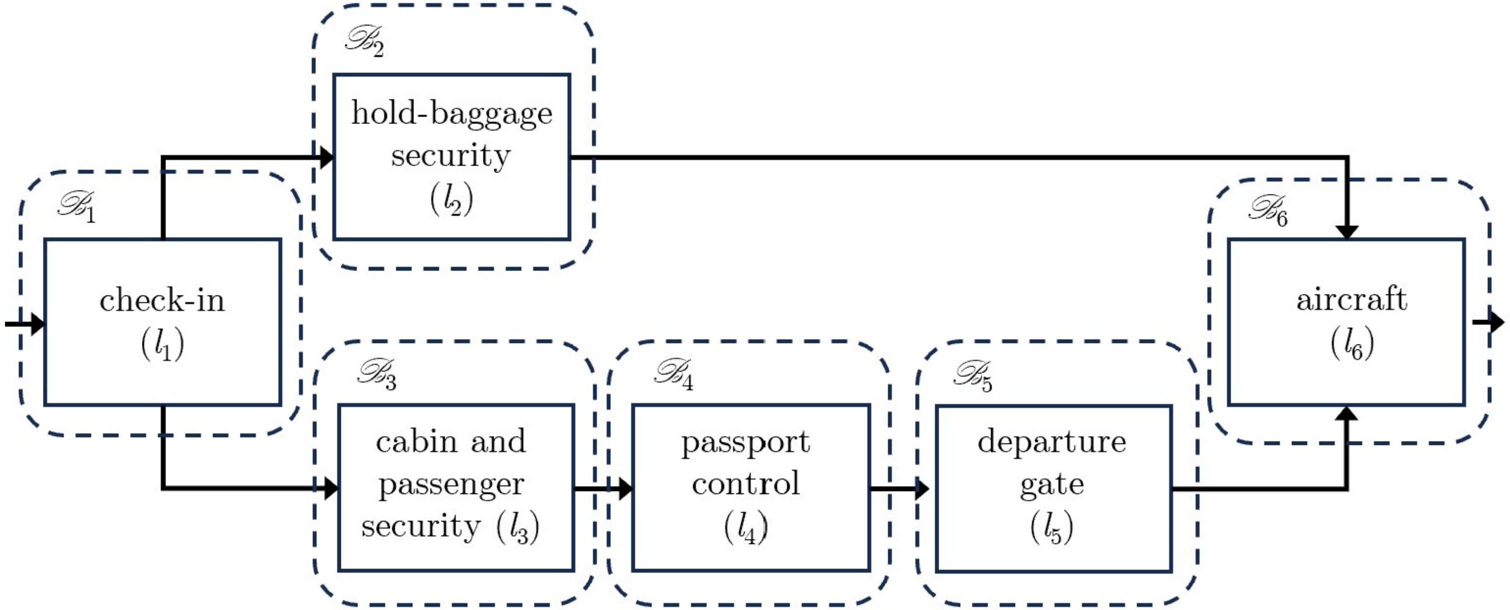
Modelling with Proof-theoretic Semantics II



- describe each component C_i by a formula ϕ_i — this is its *policy*
- its model is given by a base $\mathcal{B}_i$ and resources S such that $\Vdash^{S}_{\mathcal{B}_i} \phi_i$
- model *interfacing* by a base $\mathcal{C}$ governing input/output
- construct a model $\mathcal{D}$ of the system by taking the union of the components, $\mathcal{D} := \mathcal{B}_1 \cup \ldots \cup \mathcal{B}_n \cup \mathcal{C}$

Remark. This approach to modelling is both *compositional* and *substitutional*.

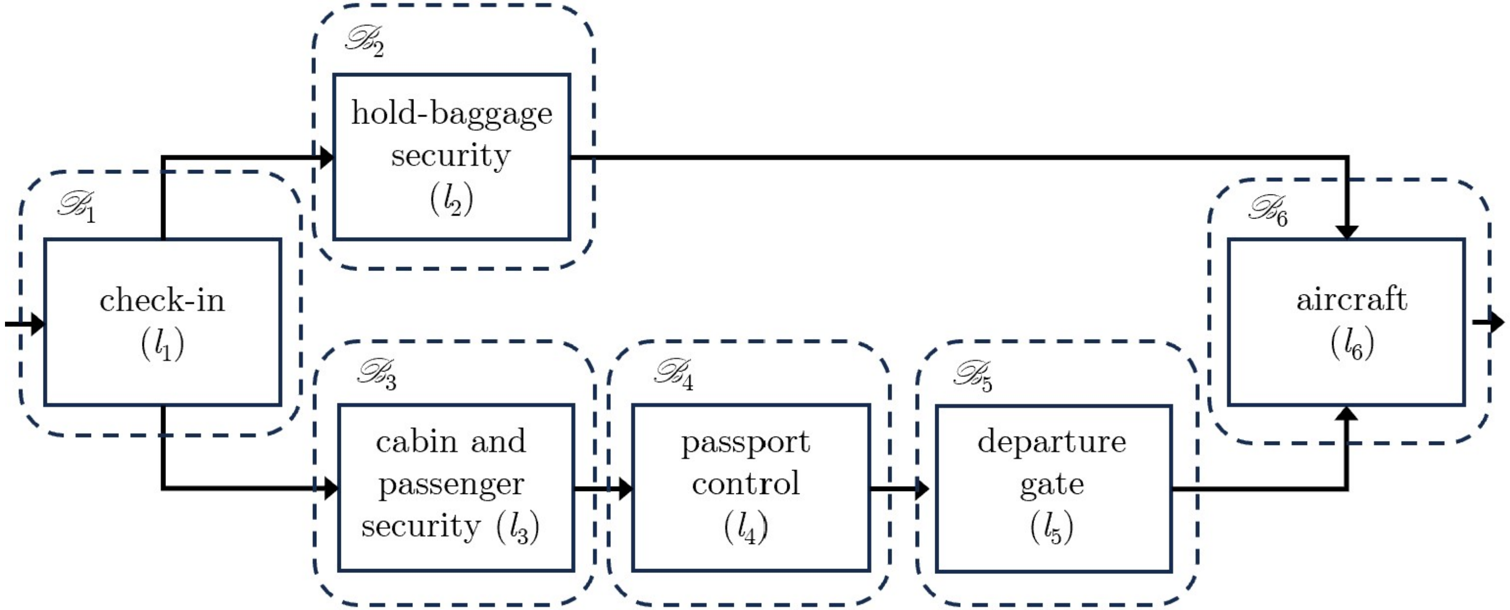
Example: Airport Security



Example: Airport Security modelled in BI

- You arrive at *check-in* (l_1). You show your passport, receive your boarding-card, and drop your hold-baggage.
- This situation is described by $\phi_1 := p \multimap ((p \land t) * h)$ — atom p denotes your passport, t denotes the boarding-card (ticket), and h denotes the baggage-label.
- The $*$ is used because the system bifurcates at this point and the resources $p \land t$ and h go to *separating* components, and the $\multimap$ is used because the system is modified: the state of passport p is changed as it only goes down one branch (the same as your ticket, t) and is no longer globally available.
- An inferential model is given by a base $\mathcal{B}_1$ supporting ϕ_1.

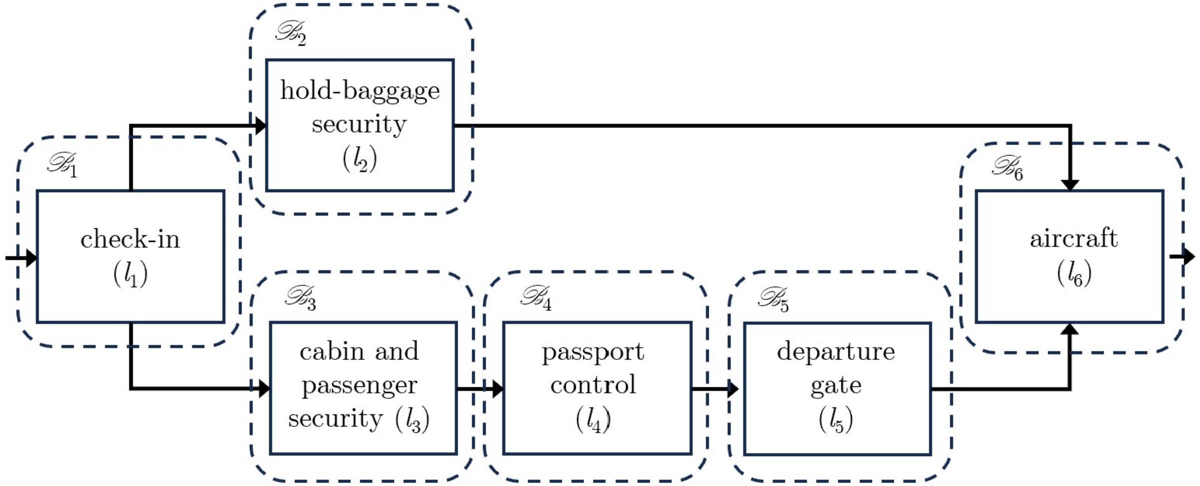
Example: Airport Security



Example: Airport Security modelled in BI

- Next, two processes occur in parallel, one through l_2 and one through l_3, l_4, l_5.
- The top path of the picture passes through *hold-baggage security* (l_2). Here, the label h on your baggage is verified (and the baggage itself is checked). That h validates is denoted by $\phi_2 := h \multimap s_{\text{hold}}$.
- The bottom path of the picture passes through l_3, l_4, and l_5, modelled by ϕ_3.
- Clearly, we could give a little more detail and model l_3, l_4, and l_5 separately. No resources are consumed on this path, so progression between locations would be handled by $\supset$.

Example: Airport Security modelled in BI



- **Resources:** p (passport), t (ticket/card), h (hold-baggage), s_{hold} (security certificate), and s_{cabin} (security certificate)
- **Component Policies for l_1, l_2, l_3:** $\phi_1 = p \mathrel{-\!\!*} ((p \land t) * h)$, $\phi_2 = h \mathrel{-\!\!*} s_{\mathrm{hold}}$, and $\phi_3 = t \supset s_{\mathrm{cabin}}$
- **Combined Policy:** $\phi = \phi_1 \mathrel{-\!\!*} (\phi_2 * \phi_3)$

Arriving with a valid ticket t and passport p is modelled by $\mathcal{B}$ such that $\Vdash_{\mathcal{B}}^{p \,\fatsemi\, t} \phi$.

And finally ...

- ... you clear passport control, get resource s_{pass}
- Eventually, you arrive at the gate (with no money, as the airport has probably persuaded you to spend it all in its shops).
- Your ticket and passport are checked, and you are granted access:

$$(s_{cabin} \wedge p \wedge t) \supset s_{gate}$$

- You board the aircraft. Before it can depart, passengers and hold-baggage must be reconciled. For each passenger, the separate certificates must be combined:

$$s_{gate} * s_{hold} \mathrel{-\!\!*} \mathit{flight}$$

For more details, see the MFPS 2024 paper, *Inferentialist Resource Semantics*.

Thesis

The paradigm of 'proof-theoretic semantics' provides an account of resource semantics that uniformly encompasses both the number-of-uses and sharing/separation interpretations of logics.

Additional References

- Alexander Gheorghiu, Tao Gu, and David Pym. Proof-theoretic Semantics for Intuitionistic Multiplicative Linear Logic. *Studia Logica*, 2024. https://doi.org/10.1007/s11225-024-10158-6.
- Alexander Gheorghiu, Tao Gu, and David Pym. Inferentialist Resource Semantics. In *Proc. Mathematical Foundations of Programming Semantics* (MFPS), Oxford, June 2024. Electronic Notes in Theoretical Informatics and Computer Science (ENTICS). `https://arxiv.org/pdf/2402.09217`.
- Alexander Gheorghiu, Tao Gu, and David Pym. Proof-theoretic-semantics for the Logic of Bunched Implications. Submitted, 2024. `https://arxiv.org/abs/2311.16719`.
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Additional References

- S. Ishtiaq and P. O'Hearn. BI as an Assertion Language for Mutable Data Structures. *Proc. ACM POPL*, 2001.
- P. O'Hearn and D. Pym. The Logic of Bunched Implications. *Bulletin of Symbolic Logic* 5(2), 215–244, 1999.
- Alexander Gheorghiu and David Pym. Semantical Analysis of the Logic of Bunched Implications. *Studia Logica* (2023). `https://link.springer.com/article/10.1007/s11225-022-10028-z`.
- Yves Lafont. Introduction to Linear Logic. *TEMPUS Summer School on Algebraic and Categorical Methods in Computer Science (Lecture Notes)*. Brno, Czech Republic, 1993.
- J.-Y. Girard. LINEAR LOGIC : ITS SYNTAX AND SEMANTICS. `https://girard.perso.math.cnrs.fr/Synsem.pdf`.

Additional References

- Timo Eckhardt and David Pym. Base-extension Semantics for S5 Modal Logic. In press, *Logic Journal of the IGPL*, 2025. Manuscript (extended version).
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Summary

So, what have we covered in 'Four Lectures on Proof-theoretic Semantics'?

- Inferentialism and Dummett-Prawitz P-tV.
- B-eS: basics, categorical interpretation, other connections.
- Reductive logic and its semantics through P-tV.
- B-eS for substructual logics and applications in system modelling.

And what have we not covered? A great deal, including:

- Lots of foundational and philosophical questions.
- Modal logic — classical, intuitionistic, epistemic.
- Full linear logic.
- Predicate logics.

Questions?

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