Errata and Remarks for *The Semantics and Proof Theory of the Logic of Bunched Implications* http://www0.cs.ucl.ac.uk/staff/D.Pym/BI-monograph-errata.pdf

David J. Pym
University College London

1 January, 2022

Abstract

Here we record the errors, typographical or otherwise, known to-date in the following publication:

> David J. Pym. The Semantics and Proof Theory of the Logic of Bunched Implications.
> Volume 26, Applied Logic Series,
> Kluwer Academic Publishers, Dordrecht/Boston/London
> Hardbound, ISBN 1-4020-0745-0, July 2002, 338 pp.
> EUR 115.00 / USD 127.00 / GBP 79.00.

This monograph, which is derived from early notes on the Logic of Bunched Implications (BI), contains may bugs, errors, and incomplete analyses. It should be considered of historical interest only. Subsequent work by many authors has greatly improved our understanding of BI.

We also present any clarifying remarks found useful to-date. We provide abstracts, citations, and errata and remarks for our related publications up to 2004. Many papers have improved our understanding of BI since then.

Introduction

We present the errors, typographical or otherwise, known to date in the following publication:

> David J. Pym. The Semantics and Proof Theory of the Logic of Bunched Implications.
> Volume 26, Applied Logic Series,
> Kluwer Academic Publishers, Dordrecht/Boston/London
> Hardbound, ISBN 1-4020-0745-0, July 2002, 338 pp.
> EUR 115.00 / USD 127.00 / GBP 79.00.

This monograph provides an account of the model theory, proof theory and computational interpretations of **BI**, the logic of bunched implications, which freely combines intuitionistic logic and multiplicative intuitionistic linear logic. Starting, on the one hand, from elementary observations about modelling resources and, on the other, from a desire to develop a system of logic within which additive (or extensional) and multiplicative (or intensional) implications co-exist with equal logical status, we give natural deduction, lambda-calculi, sequent calculus, categorical semantics, Kripke models, topological models, logical relations and computational interpretations for both propositional and predicate **BI**, within which both additive and multiplicative quantifiers also co-exist. This monograph will be of interest to graduate students and researchers in mathematical logic, philosophical logic, computational logic and theoretical computer science.

We also present any clarifying remarks found useful to-date. We include the citation details and abstracts of recent publications which extend the material of this monograph.

Errata

Introduction

1. p. xxx, l. 2: insert "and" between "Weakening" and "Contraction".

Part I

Chapter 2: Natural Deduction for Propositional BI

1. p. 16, Table 2.1: the conclusion of the $\bot(E)$ rule should be $\Delta(\Gamma) \vdash \phi$.
2. p. 19, l. -6: t should be M.
3. p. 20, Table 2.2: the conclusion of the $\bot(E)$ rule should be $\Delta(\Gamma) \vdash \bot_\phi(M) : \phi$.
4. p. 23, l. 20: insert "in **NBI** without Cut" between "admissble" and ":".
5. p. 23: after the rule at the foot of the page, add "where $\Gamma(\Delta_1 \mid \ldots \mid \Delta_m)$ denotes a bunch with multiple distinct sub-bunches,".
6. p. 27, l. -5: the premiss of the rule should be $\Gamma \vdash \alpha x : \phi.M : \phi \to \psi$.
7. p. 28, l. 2: the premiss of the rule should be $\Gamma \vdash \lambda x : \phi.M : \phi \mathrel{-\!\!*} \psi$.

Chapter 4: Kripke Semantics

1. p. 60, l. 1, 2: delete "extension of bunches" and "semi-colon, ";"", and replace "of the form $\Gamma; \Delta$" with "such that $\phi_{\Gamma'} \vdash \phi_\Gamma$". This is an embarrassing remnant of an earlier, faulty formulation. This correction has no known consequences. Thanks to Daniel Méry and Didier Galmiche for pointing it out.
2. p. 62, l. 3: after " ... ϕ_m", insert "such that $\Gamma \vdash \phi_i$, for each i". This is an embarrassing remnant of an earlier, faulty formulation. This correction has no known consequences. Thanks to Daniel Méry and Didier Galmiche for pointing it out.

Chapter 6: Propositional BI as a Sequent Calculus

1. p. 90, Table 6.1: the $\vee L$ rule is mis-stated: It should be

$$\frac{\Gamma(\phi) \vdash \chi \quad \Gamma(\psi) \vdash \chi}{\Gamma(\phi \vee \psi) \vdash \chi} \quad \vee L.$$

   This correction also applies to the statement of $\vee L$ in Definition 1 of [AP01] and Definition 5 of [GMP02] (see "Remarks", below). This correction has no known consequences.

2. p. 91: The proof-sketch for Cut-elimination is not adequate. A much stronger are is required than is suggested here. Proofs of Cut-elimination for BI have been provided by Brotherston (Bunched Logics Displayed, Studia Logica 100(6), Dec 2012) and Gheorghiu and Marin (Focused Proof-search in the Logic of Bunched Implications, Proc. FoSSaCS 2020, LNCS 12650, Springer, 2020).

Chapter 7: Towards Classical Propositional BI

1. p. 101, Table 7.1: the $\vee L$ rule is mis-stated: It should be

$$\frac{\Gamma(\phi) \vdash \Delta \quad \Gamma(\psi) \vdash \Delta}{\Gamma(\phi \vee \psi) \vdash \Delta} \quad \vee L.$$

   This correction has no known consequences.

2. p. 101, Table 7.1: the $\wedge R$ rule is mis-stated: It should be

$$\frac{\Gamma \vdash \Delta(\phi) \quad \Gamma \vdash \Delta(\psi)}{\Gamma \vdash \Delta(\phi \wedge \psi)} \quad \wedge R.$$

   This correction has no known consequences.

3. p. 100, l.18: insert 'provided the negations and the units are excluded' after Cut-elimination.

Chapter 8: Bunched Logical Relations

1. p. 114, l. 19: replace "$\sqsubseteq$ is extension of bunches by semi-colon, ";" " with "$\Gamma' \sqsubseteq \Gamma$ if $\phi_\Gamma \vdash \phi'_\Gamma$". This is an embarrassing remnant of an earlier, faulty formulation. Thanks to Daniel Méry and Didier Galmiche for pointing it out.

Part II

Chapter 10: Introduction to Part II

1. p. 149, l. -11: missing premiss in $\exists I$: $X \vdash \Gamma : \mathbf{Prop}$.

Chapter 11: The Syntax of Predicate BI

1. p. 160, Table 11.1: the conclusion of the $\bot(E)$ rule should be $Y(X) \vdash_\Sigma \bot_\phi(t) : \phi$.

Chapter 12: Natural Deduction and Sequent Calculus

1. p. 164, l. 7: replace "where $X \vdash_{\Sigma,\Xi} \Gamma : \mathbf{Prop}$ and $X \vdash_{\Sigma,\Xi} \phi : \mathbf{Prop}$." with "where $X_1 \vdash_{\Sigma,\Xi} \Gamma : \mathbf{Prop}$ and $X_2 \vdash_{\Sigma,\Xi} \phi : \mathbf{Prop}$ and $X = X_1, X_2$ or $X = X_1; X_2$."

2. p. 166, Table 12.1: the conclusion of the $\bot(E)$ rule should be $(Y(X))\Delta(\Gamma) \vdash_{\Sigma,\Xi} \phi$.

3. p. 167, l. 7: replace "then $X \vdash_{\Sigma,\Xi} \phi :$ **Prop**" with "then $X_1 \vdash_{\Sigma,\Xi} \Gamma :$ **Prop** and $X_2 \vdash_{\Sigma,\Xi} \phi :$ **Prop**, where $X = X_1, X_2$ or $X = X_1; X_2$"
4. p. 169, l. 9: missing premiss in $\exists I$: $X \vdash_{\Sigma,\Xi} \Gamma :$ **Prop**.
5. p 169, l. 9: missing premiss in $'existsI$: $X \vdash_{\Sigma,\Xi} \Gamma :$ **Prop**

Chapter 13: Kripke Semantics for Predicate BI

1. p. 180, l. 14: $=$ should be $\subseteq$. See Remarks, below.
2. p. 180, l.-14: $\in$ should be $\subseteq$ (but see Footnote 1).
3. p. 182, l. 7: Footnote 2 has been omitted from the bottom of the page. The missing text is the following: We could also go directly to $X \cong Y \cong Z$, *i. e.*, building in a "Contraction".
4. p. 188, l. 11: replace "$(X)\Gamma \vdash_{\Sigma,\Xi} \phi$" with $(X)u \mid m \models^{\mathcal{M}}_{\Sigma,\Xi} \Gamma, \phi$".
5. p. 188, l. 13,14: delete ", with $X' \vdash_{\Sigma,\Xi} \phi_\Gamma :$ **Prop** and $X'' \vdash_{\Sigma,\Xi} \phi :$ **Prop**.
6. p. 196, l. 11: replace "extension" with "derivability" and delete "by semi-colon, ";" ". This is an embarrassing remnant of an earlier, faulty formulation. Thanks to Daniel Méry and Didier Galmiche for pointing it out.

Remarks

Part I

Natural Deduction for Propositional BI

1. p. 20, Table 2.2: although the Cut rule is included in Table 2.2, **NBI** for $\alpha\lambda$ is, of course, normally intended without Cut (which is then admissible).

Chapter 4: Kripke Semantics

1. p. 53, Definition 4.1, *et seq.*: since we take $[\![-]\!]$ to be a partial function, so that not all propositions need be interpreted at all worlds, it should be made clearer than is stated that all constructions dependent on $[\![-]\!]$ are conditional upon definedness.
2. p. 61, l. 14,15,16: The sentence begining "Henceforth, ..." is less clear than intended. The point is that, at each $\vee$-reduction in the construction of the monoid, a choice of disjunct must be made. Hence our use of just Γ, *etc.*, in what follows. The other choice is maintained to provide a record of the other possible choice, and so give a construction which is uniform with that required for the corresponding lemma in Chapter 5. (I am particularly grateful to Didier Galmiche and Daniel Méry for drawing my attention to this lack of clarity.)

Chapter 5: Topological Kripke Semantics

1. p. 71 Definition 5.4, *et seq.*: since we take $[\![-]\!]$ to be a partial function, so that not all propositions need be interpreted at all worlds, it should be made clearer than is stated that all constructions dependent on $[\![-]\!]$ are conditional upon definedness.

Part II

Chapter 11: The Syntax of Predicate BI

1. p. 160, Table 11.1: although the Cut rule is included in Table 11.1, **NBI** for $\alpha\lambda$ is, of course, normally intended without Cut (which is then admissible).

Chapter 13: Kripke Semantics for Predicate BI

1. p. 180, Definition 13.2, Definition 13.3, *et seq.*: since we take $[\![-]\!]$ to be a partial function, so that not all propositions need be interpreted at all worlds, it should be made clearer than is stated that all constructions dependent on $[\![-]\!]$ are conditional upon definedness.

2. p. 180, l. 14: $[\![x \in A]\!]^D \subseteq D$ means, of course, simply that the interpretation of each variable at each world m is a subset of $D(m)$.

3. p. 180, Footnote 1: "$D(m)$" is misleading. "each $[\![x \in A]\!]^D(m)$" would be clearer. The collection of such singletons would then amount to the base case of Definition 13.3.

4. p. 185, l. 5: it should be noted that the observed consistency applies only for the language with a single type. I am grateful to Bodil Biering and Lars Birkedal for pointing out that this should be emphasized.

5. p. 197, l. 9,10,11: See the remark above for p. 61, l. 14,15,16.

6. Items 3, 4, and 5 in the errata for Chapter 12 and 4, 5 in Chapter 13, whilst necessary to fix a technical problem, leave us in a conceptually and computationally undesirble situation: we have no clean characterization of a useful notion of well-formedness, leading to obvious computational difficulties in, say, theorem proving. We conclude that such a highly multiplicative system is far too complex, at least in its current formulation, to be of much value. All of the applications of multiplicative quantifiers known to-date require much simpler systems. It is the author's recommendation that readers should not devote very much of their resources to studying the system of predicate BI discussed in these chapters. I am grateful to Bodil Biering and Lars Birkedal for raising these issues in an appendix of Biering's Master's dissertation. The reference [Pym04] therein refers to a previous version of this list of errata and remarks, replaced by the present version.

Chapter 14: Topological Kripke Semantics for Predicate BI

1. p. 201, Definition 14.1, *et seq.*: since we take $[\![-]\!]$ to be a partial function, so that not all propositions need be interpreted at all worlds, it should be made clearer than is stated that all constructions dependent on $[\![-]\!]$ are conditional upon definedness.

Further papers

The following papers provide immediate extensions of the material presented in the monograph.

GMP02 D. Galmiche, D. Méry and D. Pym. Resource Tableaux (extended abstract). Proc. Int. Conference on Computer Science Logic, CSL'02, Edinburgh, Scotland, September 2002. LNCS 2471, 183–199, 2002.

The logic of bunched implications, **BI** , provides a logical analysis of a basic notion of resource which has proved rich enough to provide a pointer logic semantics for programs which manipulate mutable data structures. We develop a theory of semantic tableaux for **BI**, thereby providing an elegant basis for efficient theorem proving tools for **BI**. It is based on the use of an algebra of labels for **BI**'s tableaux to solve the resource-distribution problem, the labels being the elements of resource models. In the case of **BI** with inconsistency, the challenge consists in dealing with**BI**'s Grothendieck topological models within such a based-on labels proof-search method. In this paper, we prove soundness and completeness theorems for a resource tableaux method T**BI** with respect this semantics and provides a way to build countermodels from so-called dependency graphs. As consequences, we have two new results for **BI**: the decidability for propositional **BI** and the finite model property with respect to Grothendieck topological semantics. In addition, we propose a new resource semantics by considering a partially defined monoid, which generalizes the semantics of **BI** pointer logic and is complete for **BI**.

POY04 D. Pym, P. O'Hearn and H. Yang. Possible Worlds and Resources: The Semantics of **BI**. *Theoretical Computer Science* 315(1): 257–305.

The logic of bunched implications, **BI**, is a substructural system which freely combines an additive (intuitionistic) and a multiplicative (linear) implication via bunches (contexts with two combining operations, one which admits Weakening and Contraction and one which does not). **BI** may be seen to arise from two main perspectives. On the one hand, from proof-theoretic or categorical concerns and, on the other, from a possible-worlds semantics based on preordered (commutative) monoids. This semantics may be motivated from a basic model of the notion of resource. We explain **BI**'s proof-theoretic, categorical and semantic origins. We discuss in detail the question of completeness, explaining the essential distinction between **BI** with and without bottom (the unit of or). We give an extensive discussion of **BI** as a semantically based logic of resources, giving concrete models based on Petri nets, ambients, computer memory, logic programming, and money.

Erratum: p. 285, l. -12: ", for some $P', Q = P; P'$" should be "$P \vdash Q$".

AP01 P. Armelín and D. Pym. Bunched Logic Programming (Extended Abstract). Proc. IJCAR 2001, Siena. LNAI 2083, 289-304, 2001.

We give an operational semantics for the logic programming language `BLP`, based in the hereditary Harrop fragment of the logic of bunched implications, **BI**. We introduce **BI**, explaining the account of the sharing of resources built into its semantics, and indicate how it may be used to give a logic programming language. We explain that the basic input/output model of operational semantics, used in linear logic programming, wil not work for bunched logic. We show how to obtain a complete, goal-directed proof theory for hereditary Harrop **BI** and how to reformulate the operational model to account for the interaction between multiplicative and additive structure. We give a prototypical example of how the resulting programming language handles, in contrast with Prolog, sharing and non-sharing of resources purely logically.

Errata

(a) p. 295, l. 9: the $\vee L$ should be

$$\frac{\Gamma(\phi) \vdash \chi \quad \Gamma(\psi) \vdash \chi}{\Gamma(\phi \vee \psi) \vdash \chi} \quad \vee L.$$

(b) p. 300, l. 4: the conclusion of the Unit^{ii} should begin

$$\frac{}{\langle \Gamma \mid \boxed{\phi}^{-} :: s\rangle ...} \quad \text{Unit}^{ii}.$$

(c) p. 300, l. 6: in the left-hand premiss of the $\to L^{\dagger}$ rule, m should be n.

Acknowledgements. I am grateful to all who have kindly pointed out errors or obscurities in these works.