A summary of the more recent papers on the completeness of the Logic of Bunched Implications

David Pym
UCL & Institute of Philosophy
University of London

david.pym@sas.ac.uk and d.pym@ucl.ac.uk
https://www.cantab.net/users/david.pym/

The history of attempts to give completeness results for the Logic of Bunched Implications (BI) is quite complex. The book *The Semantics and Proof Theory of the Logic of Bunched Implications* (D. Pym, 2002, Springer, with some errata at https://extras.springer.com/?query=978-1-4020-0745-3) contains early rough notes with many errors and should not be regarded as a primary source.

The following, in reverse chronological order, are the main sources:

1. Alexander Gheorghiu, Tao Gu, and David Pym. Proof-theoretic Semantics for the Logic of Bunched Implications. *Studia Logica*, 2025, doi.org/10.1007/s11225-025-10202-z.

   This is perhaps the most natural complete semantics for BI. It employs 'base-extension semantics'.

2. Alexander Gheorghiu and David Pym. Semantical Analysis of the Logic of Bunched Implications. *Studia Logica*, 2023, doi.org/10.1007/s11225-022-10028-z.

   This is a quite general treatment of completeness, employing a novel proof technique, and including an extensive discussion of the issues around completeness and its failures.

3. Several papers by Docherty and Pym explore similar issues for logics in the BI family and obtain useful completeness theorems:

   (a) Simon Docherty and David Pym. Modular Tableaux Calculi for Separation Theories. In: *Proc. FoSSaCS 2018*, LNCS 10803, Springer, 2018

   (b) Simon Docherty and David Pym. Intuitionistic Layered Graph Logics: Semantics and Proof Theory. *Logical Methods in Computer Science*, https://lmcs.episciences.org/4942.

4. D.J. Pym, P.W. O'Hearn, and H. Yang. Possible worlds and resources: the semantics of BI. *Theoretical Computer Science* 315(1): 257-305, 2004.

   This paper includes the completeness result for the Grothendieck topological version of the monoidal semantics, which is not the elementary monoidal semantics.