Paper 11
Urban Economics

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Increasing returns to city size

- We have seen that our model has two sectors, industry and services
- Industry is assume to have a competitive market structure with constant returns to scale
- Services are under monopolistic competition with internal increasing returns to scale

Increasing returns to city size

- outcome is the nonlinear relationship between the level of industry output (*Q*) and city size (*N*)
- Bigger cities are more productive
- the same relationship is also an outcome of a different tradition in urban and regional economics, that which has been strongly influenced by people such as Keynes and Kaldor
- Fingleton B (2001) 'Equilibrium and economic growth : spatial econometric models and simulations' *Journal of Regional Science*, 41 117-148
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Increasing returns to city size

$$I = [\sum_{t=1}^{x} i(t)^{1/\mu}]^{\mu} = [xi(t)^{1/\mu}]^{\mu} = x^{\mu} i(t)$$

$$Q = M^{\beta} I^{1-\beta}$$

M is industry labour
N is total labour = city size

$$Q = \phi N^{\gamma}$$

Output *Q* a nonlinear function
Of city size *N*

$$\gamma = 1 + (1-\beta)(\mu - 1)$$

Importance of industry labour

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Increasing returns to city size

$$Q = \phi N^{\gamma}$$

Nonlinear

↓

$$\ln Q = \ln \phi + \gamma \ln N$$

Take natural logarithms
loglinear

↓

$$\ln(Q / N) = \frac{\ln(\varphi)}{\gamma} + \left[\frac{\gamma - 1}{\gamma}\right] \ln(Q)$$

Productivity
(qua wages)
A function of city size (*Q*)

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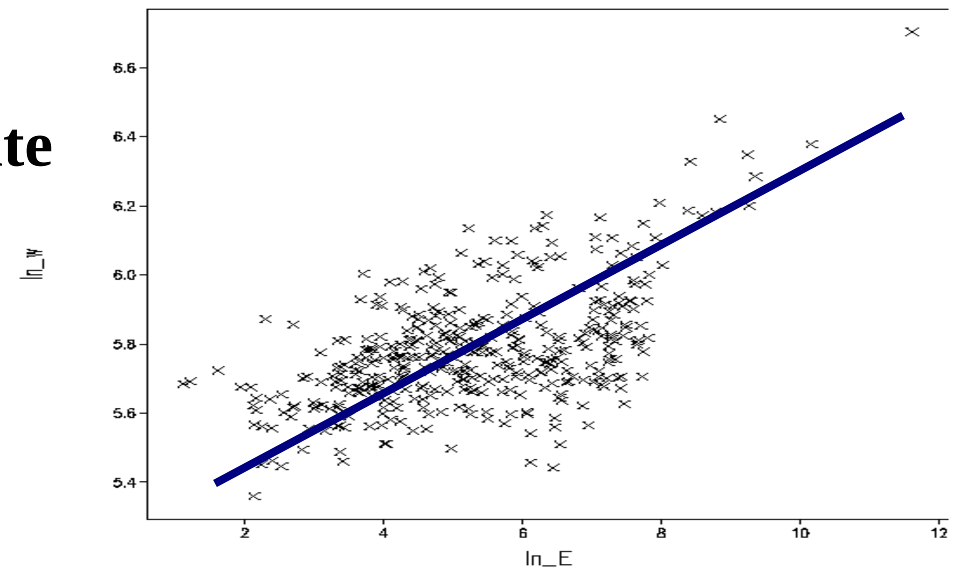
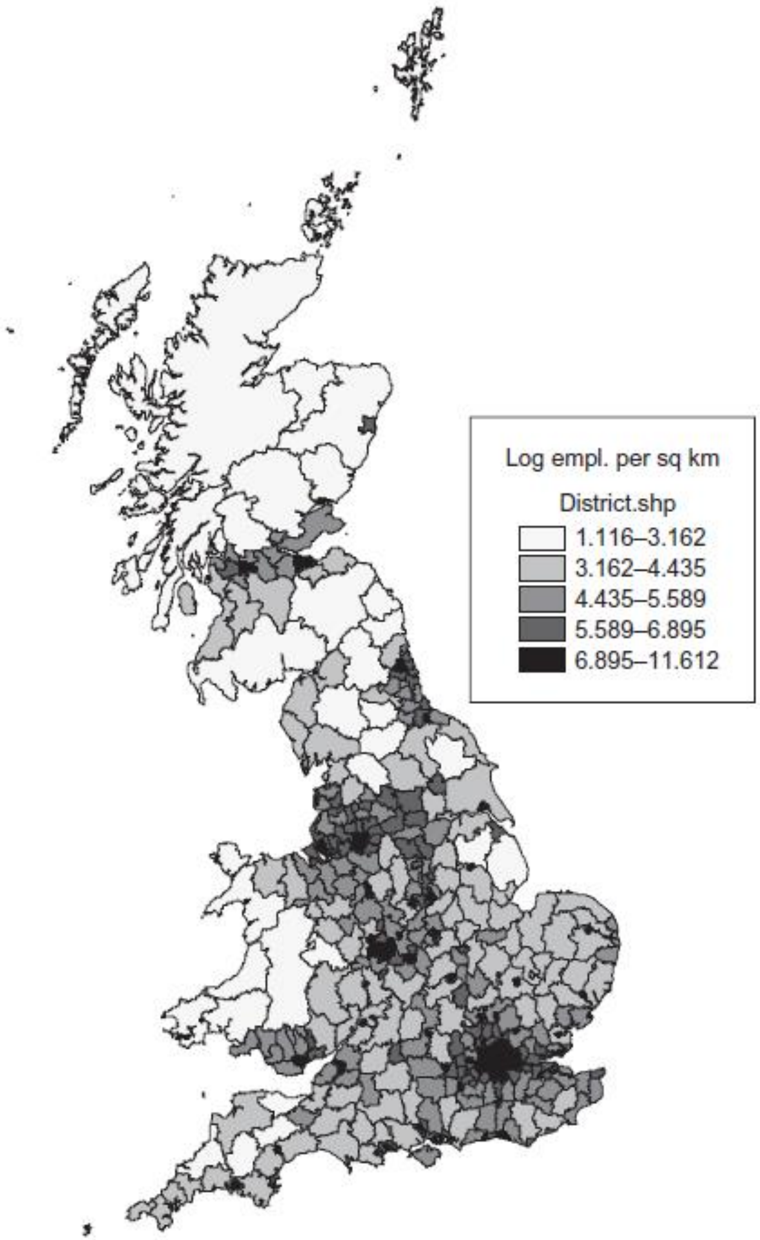
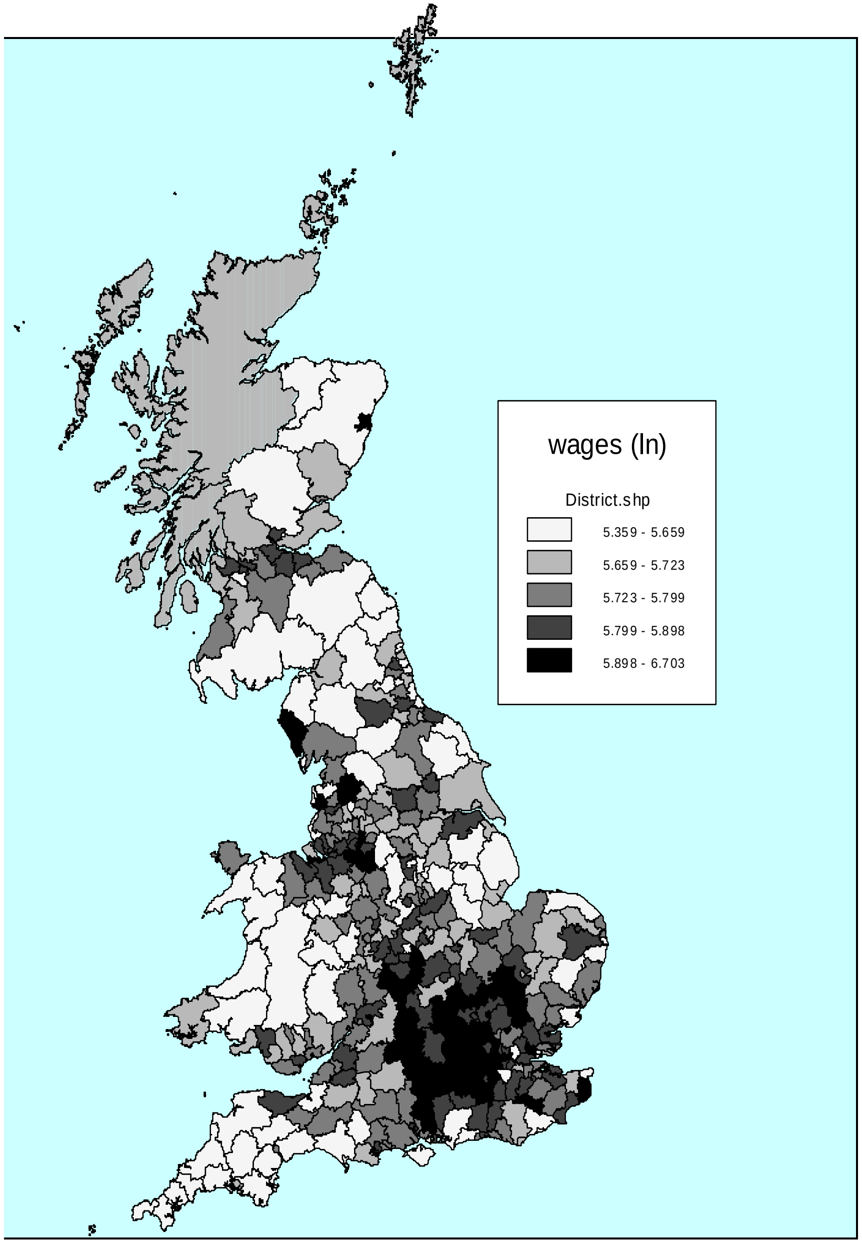
Evidence: UK local authorities

Log wage rate



Log Employment density ≈ city size





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Micro-foundations

- In order to understand how we combine the two production functions to arrive at the loglinear relationship between *Q* and *N*
- we need to examine the assumptions being made about firms' production more closely
- We need to examine the micro-foundations of our theory

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Micro-economic foundations : industry

- Commence with the industry profit function
- Industry revenue equals price on the world market *P* times amount produced *Q*
- Industry costs equal wages *w* times number of workers *M* plus the prices of each service input *p(t)* times the amount of each service used *i(t)*, added up across all *x* services

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Micro-economic foundations : industry

$$\Pi_m = P_m Q - (wM + \sum_{t=1}^{x} p_t i(t))$$

profit revenues costs

P_m is set by world market conditions and is treated as a constant

Q is subject to diminishing returns to I, so as I increases, as a result of $i(t)$ increasing, there is a less than proportionate increase in Q

$$Q = M^{\beta} I^{1-\beta}$$

The increase in costs as a result of an increase in $i(t)$ is linear

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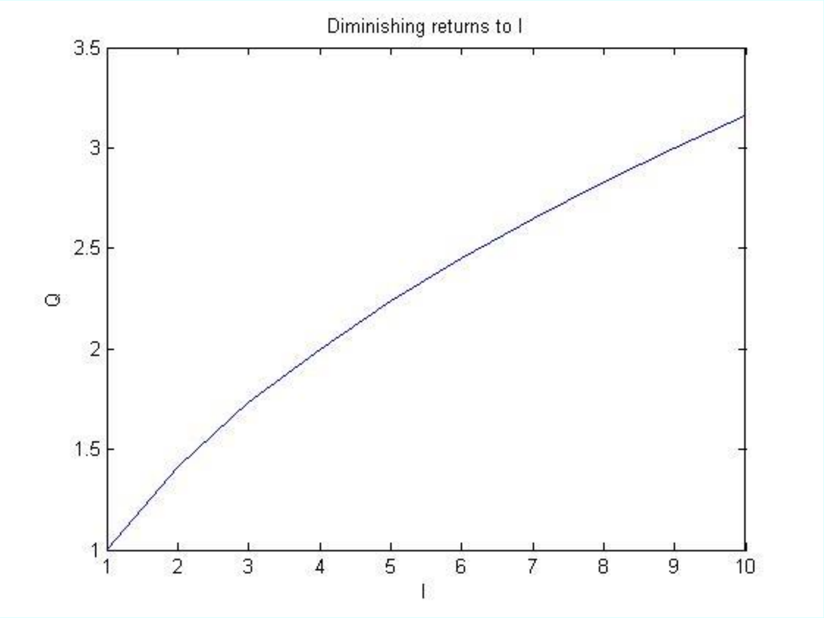
Industry Profit Maximisation

Constant returns to scale but diminishing returns to each input $\longrightarrow Q = M^{\beta} I^{1-\beta}$

Q is subject to diminishing returns to *I*, so as *I* increases, as a result of *i*(*t*) increasing, there is a less than proportionate increase in *Q*



$M = 1$

$I = 1,2,\ldots,10$

$\beta = 0.5$

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Industry Profit Maximisation

competitive sector profits Π rise to a maximum then fall as the quantity $i(t)$ increases, assuming a given price p_t . This is the outcome of profits equal to revenue minus costs, where revenue is a nonlinear function of $i(t)$ while costs are a linear function

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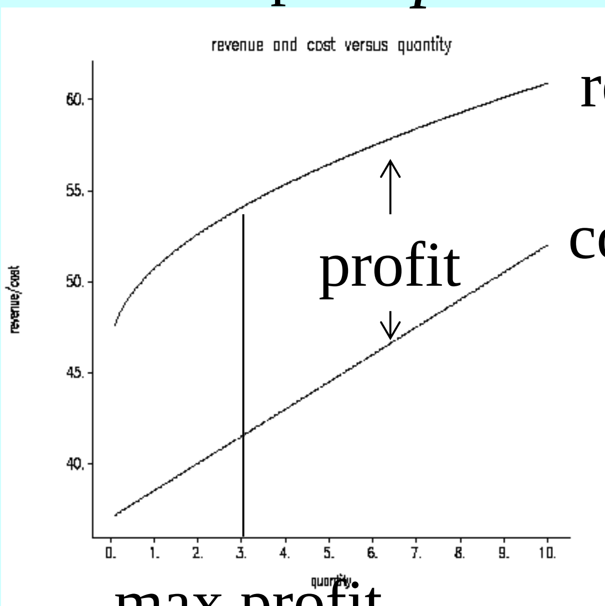
Industry Profit Maximisation

low price p_t



revenue

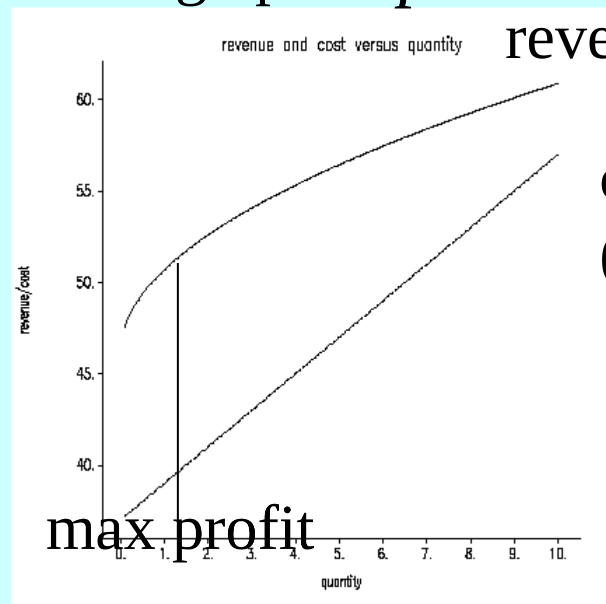
cost

profit

max profit

demand ($i(t)$) ⟶

high price p_t



revenue (same)

cost
(higher)

max profit

demand ($i(t)$) ⟶

Higher price, higher cost

Lower profit

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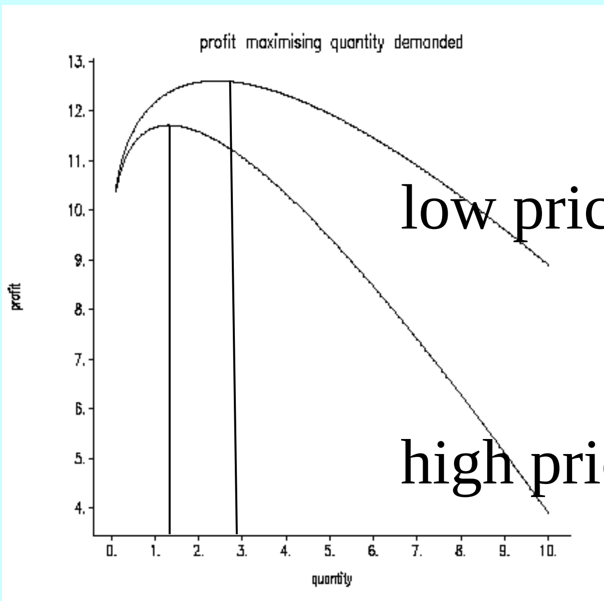
Industry Profit Maximisation

At any given price p_t there is an amount $i(t)$ which is demanded by the competitive sector that is consistent with profit maximization in that sector

Profit Π_m

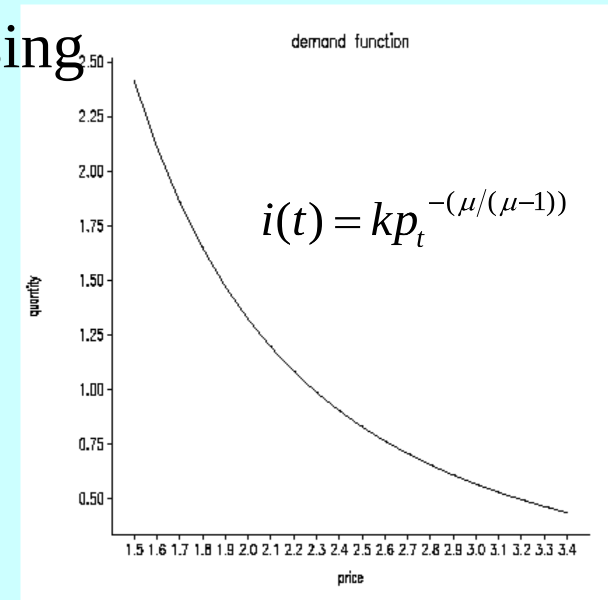


low price

high price

Demand $i(t)$

profit maximising demand $i(t)$



$$i(t) = kp_t^{-(\mu/(\mu-1))}$$

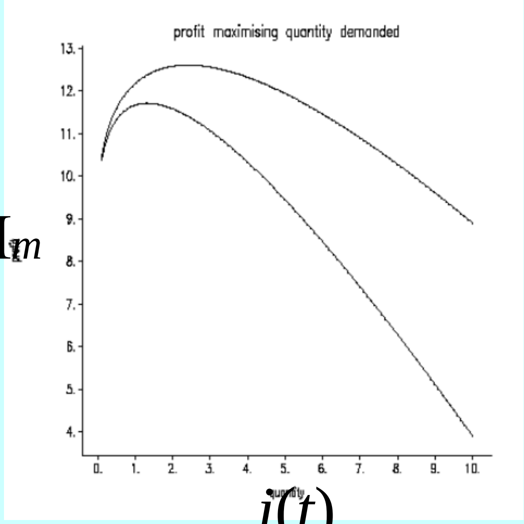
Price p_t

As price goes from low to high, profit max. demand falls

Industry Profit Maximisation

$$\frac{\partial \Pi_m}{\partial i(t)} = 0$$

Π_m



$i(t)$

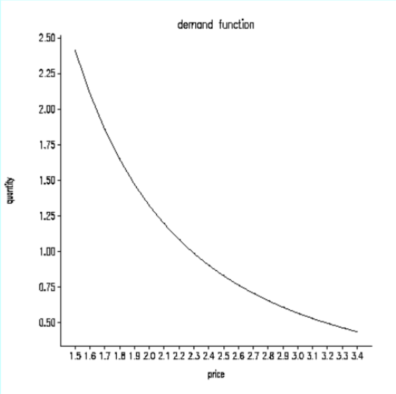
Maximum profit is where slope of profit/demand curve Falls to zero

Gives the *i(t)* at which industry Profits maximised

This clearly depends on price p_t

$$\frac{\partial \Pi_m}{\partial i(t)} = i(t) - [kp_t^{-1}]^{\frac{\mu}{\mu-1}} = 0$$

$$i(t) = kp_t^{\frac{-\mu}{\mu-1}} \longrightarrow i(t)$$



p_t

Profit maximising demand for services depends on their price
As price increases, demand falls
So we see a mathematical relationship between
Price and demand, we use this demand function subsequently

Micro-economic foundations : services

- Under the model, each firm produces a different variety
- The reason is due to the fixed costs incurred by each firm
- There is no point in splitting a variety between different firms
  - each would incur the fixed cost
  - the level of output of each firm would be smaller
  - small output means average costs higher
- Hence it is better to produce each variety in a single firm

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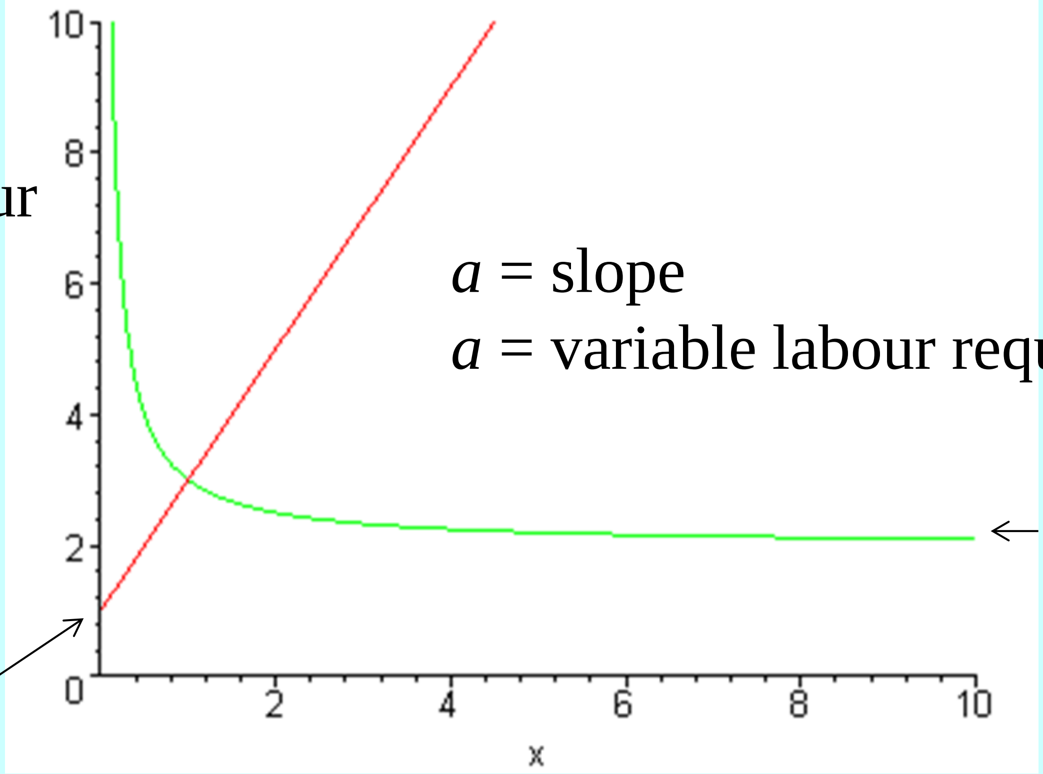
Micro-economic foundations : services

L = labour



a = slope

a = variable labour requirement

Average cost curve

s = fixed labour requirement

Output $i(t)$

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Micro-economic foundations : services

- Should firms with internal increasing returns keep increasing in size indefinitely, since bigger means better?
- Is there an equilibrium size to which they converge?
- Increasing output increases costs as well as revenues
  - Revenues increase because sales are higher, but costs increase because more workers are employed
- So, there is an equilibrium service firm size at which profits are at a maximum

Services Profit Maximisation

$$\pi = p_t i(t) - wL$$

$$\pi = p_t i(t) - w(ai(t) + s)$$

Price times quantity sold
Equals revenue

Wages times labour
Equals costs

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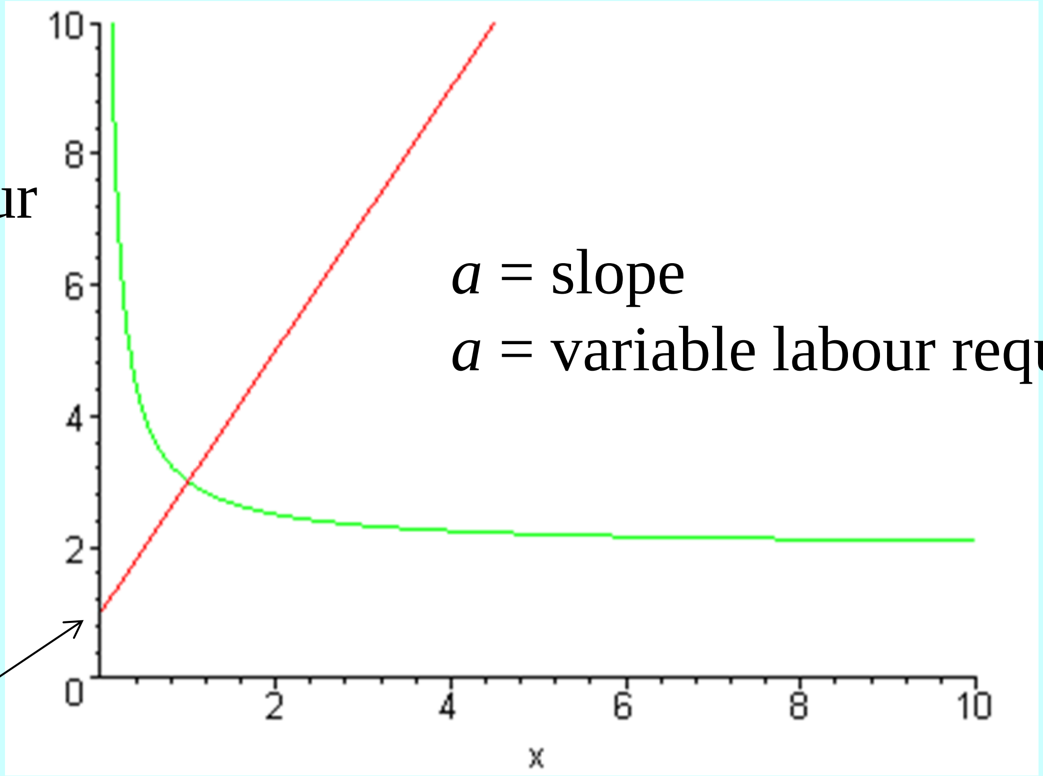
Services Profit Maximisation

L = labour



a = slope
a = variable labour requirement

s = fixed labour requirement

Output *i*(*t*)

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Services Profit Maximisation

$$L = s + ai(t)$$

s - fixed labour requirement

a – marginal labour requirement

Services Profit Maximisation

$$i(t) = kp_t^{-(\mu/(\mu-1))}$$

Demand function
(from Industry profit max.)

$$\pi = p_t i(t) - w(ai(t) + s)$$

Profit function

$$\pi = p_t k p_t^{-(\mu/(\mu-1))} - w(akp_t^{-(\mu/(\mu-1))} + s)$$

↑

Profit function after substituting for $i(t)$

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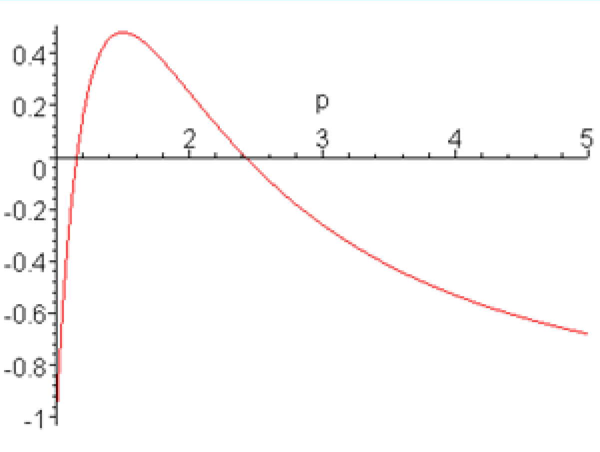
Services Profit Maximisation

$$\pi = p_t k p_t^{-(\mu/(\mu-1))} - w(akp_t^{-(\mu/(\mu-1))} + s)$$

Profit versus price, with other terms held constant

w=1,a=1,s=1, k=10, μ=1.5

p
r
o
f
i
t



price

profits are at a peak at p_t = 1.5 = $wa\mu$

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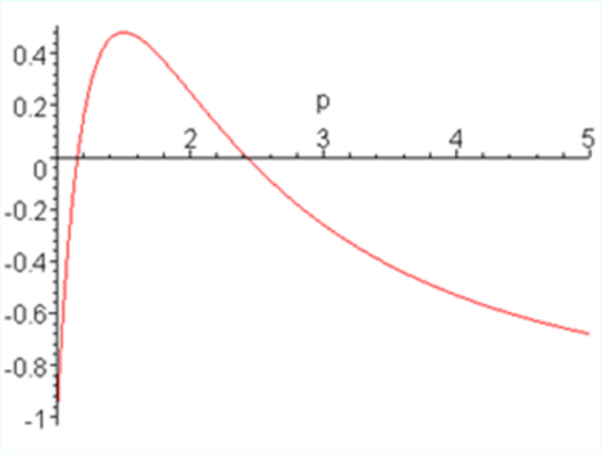
Services Profit Maximisation

$$\pi = p_t k p_t^{-(\mu/(\mu-1))} - w(akp_t^{-(\mu/(\mu-1))} + s)$$

Finding the profit maximising price

$$\frac{\partial \pi}{\partial p_t} = p_t - wa\mu = 0$$

$$p_t = wa\mu$$



Services Profit Maximisation

- Under monopolistic competition we see that prices are $wa\mu$ so not equal to marginal cost (wa) but higher.
- Since all these three terms $wa\mu$ are constants, the price charged by all firms is the same, equal to p_t, so they all have identical demand and output.
- This is known as mark-up pricing, since the marginal cost of producing an extra unit of output is wa, and the price the firm charges is a mark-up on this marginal cost by the factor μ

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Services Profit Maximisation

- We assume that there is free entry and exit from the market.
- Each new firm that enters provides a new variety of service, so that industry allocates spending over a wider and wider spectrum of services
- Each variety is a substitute for each other, so the entry of new varieties implies that existing service firms have their profits eaten into by the advent of the new varieties

Services Profit Maximisation

- This process of entry of new varieties continues until profits fall to zero. In fact the zero profit position is an equilibrium. If profits are negative then firms exit and profits rise to zero
- This zero profit situation is an equilibrium and defines the equilibrium firm size

Services Profit Maximisation

The level of output at equilibrium

With free entry under monopolistic competition, profits are Driven down (or up) to zero

$$\pi = p_t i(t) - w(ai(t) + s) = 0$$

But we know the price $$p_t = wa\mu$$

So we can substitute to get the size of the typical firm At equilibrium

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Services Profit Maximisation

$$\pi = p_t i(t) - w(ai(t) + s) = 0$$

Zero profits at equilibrium

rearrange

$$p_t i(t) = w(ai(t) + s)$$

Substitute for price

$$wa\mu i(t) = wai(t) + ws$$

rearrange

$$wa\mu i(t) - wai(t) = ws$$

rearrange

$$(\mu - 1)wai(t) = ws$$

$$i(t) = \frac{ws}{(\mu - 1)wa} = \frac{s}{a(\mu - 1)}$$

Equilibrium output

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Services Profit Maximisation

- Equilibrium output is positively related to the fixed labour requirement *s*
  - As fixed costs rise, so does the equilibrium output of the service firm
  - The reason is that higher fixed costs *s* introduces more scope for reaping scale economies, producing on a larger scale will reduce the impact of the fixed cost

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Services Profit Maximisation

- High variable labour requirements *a* reduce the output level
  - This is because increasing *a* increases the marginal cost of producing at any level of output
- Finally, increasing μ reduces the output level
  - As μ increases, we have more product differentiation, and stronger monopoly power, so the market has a large number of small producers

Increasing returns to city size

- Now we are in a position to show how merging the two production functions, one for industry and one for services, gives us the increasing returns to city size function

$$I = [xi(t)^{1/\mu}]^{\mu} = x^{\mu} i(t) \longrightarrow Q = M^{\beta} I^{1-\beta}$$

$$Q = \varphi N^{\gamma}$$

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Increasing returns to city size

- The point is that each service firm has the same output $i(t)$, since the determinants of equilibrium output s, a and μ are constants
- This means they each have the same labour force L, since

$$L = ai(t) + s$$

- so if we know the total services labour force, dividing by the constant size L of the typical firm will give us the number of firms

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Increasing returns to city size

In the Cobb-Douglas β is the share of total employment N that works in industry, that is $M = \beta N$

So 1- β is the share that works in services

So dividing total services employment (1- β)N by the size of each firm gives the number of service firms x

$$x = \frac{(1-\beta)N}{ai(t)+s}$$

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Increasing returns to city size

Industry p.f. $Q = M^{\beta} I^{1-\beta}$

Services p.f. $I = x^{\mu} i(t)$

substitute $Q = M^{\beta} (x^{\mu} i(t))^{1-\beta}$

$$Q = M^{\beta} x^{\mu-\mu\beta} i(t)^{1-\beta}$$

Find number of service firms $x = \dfrac{(1-\beta)N}{ai(t)+s}$

$$M = \beta N$$

$$Q = N^{\beta+\mu-\mu\beta} \beta^{\beta} (ai(t)+s)^{\mu(\beta-1)} i(t)^{1-\beta} (1-\beta)^{-\mu(\beta-1)}$$

Rearrange and simplify $Q = N^{\beta+\mu-\mu\beta} \varphi$

$$Q = \varphi N^{1+(1-\beta)(\mu-1)} = \phi N^{\gamma}$$

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Increasing returns to city size

$$Q = \phi N^{\gamma}$$

Nonlinear

↓

$$\ln Q = \ln\phi + \gamma \ln N$$

Take natural logarithms
loglinear

↓

$$\ln(Q/N) = \frac{\ln(\varphi)}{\gamma} + \left[\frac{\gamma - 1}{\gamma}\right]\ln(Q)$$

Productivity
(qua wages)
A function of city size (*Q*)

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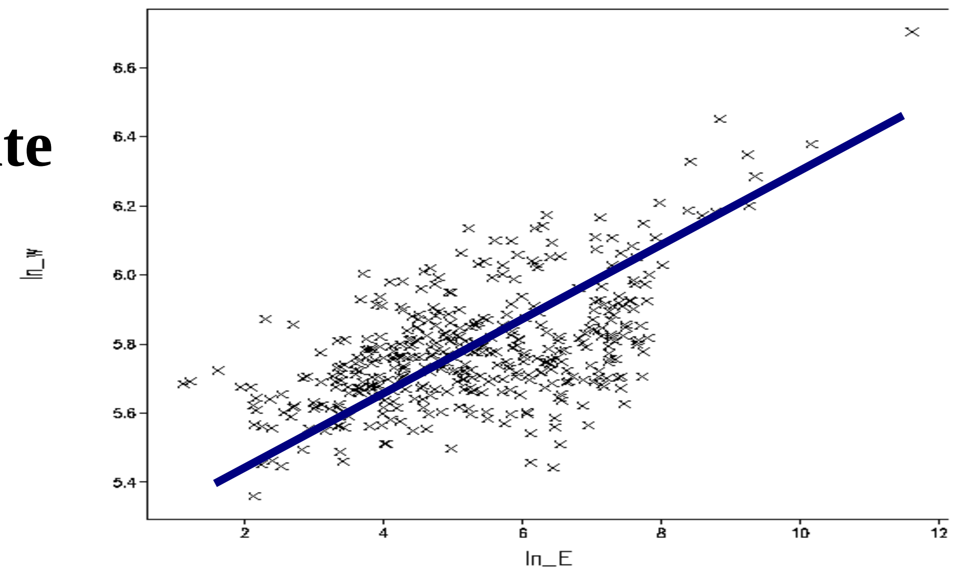
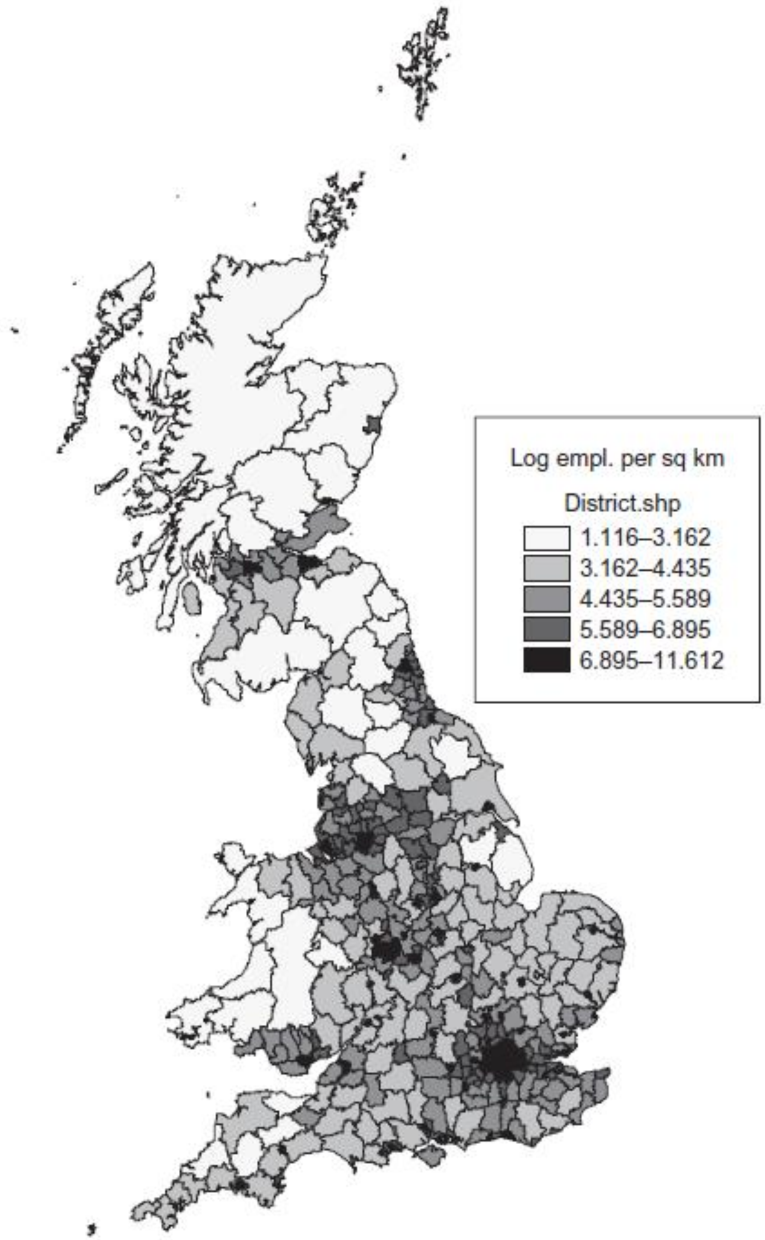
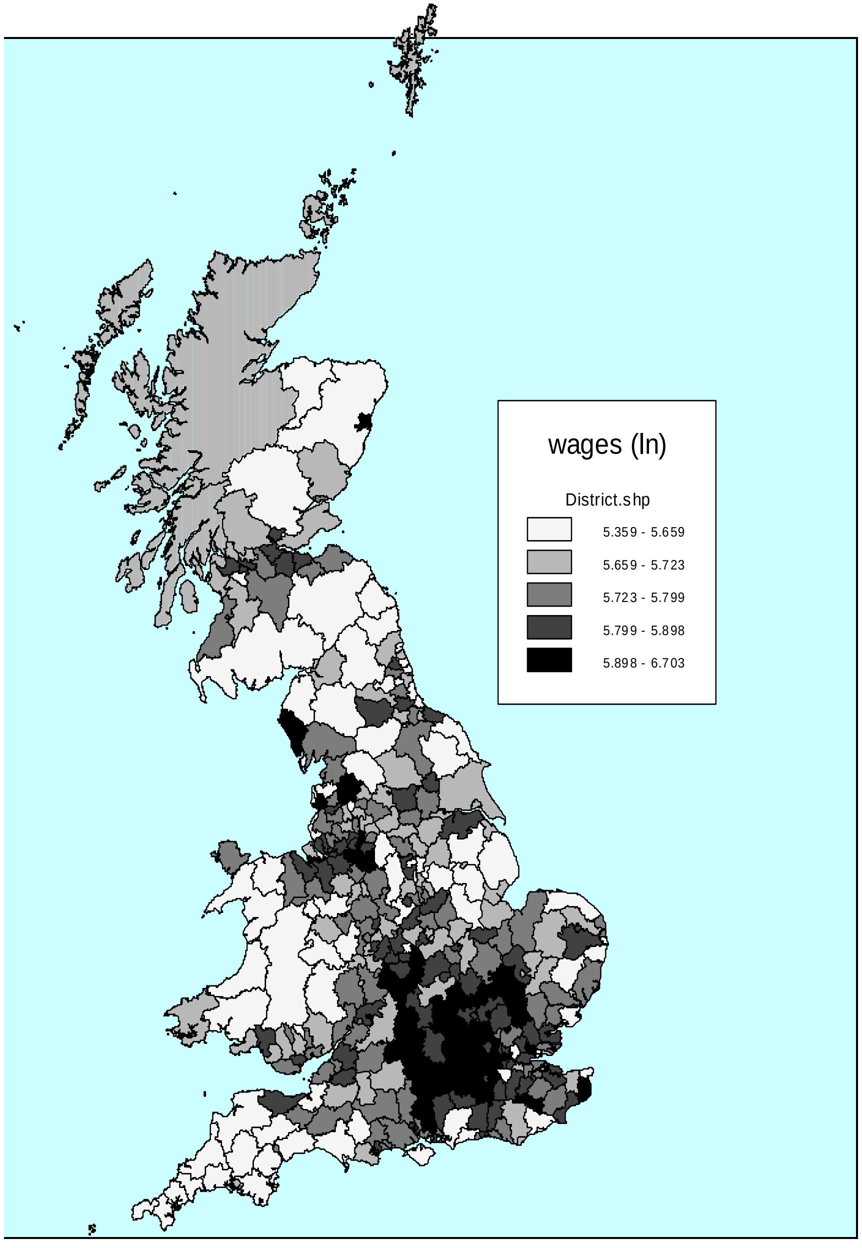
Evidence: UK local authorities

Log wage rate



Log Employment density ≈ city size





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