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Increasing returns: evidence from local wage rates in Great Britain

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This paper shows that wage rate variations among local areas of Great Britain are significantly positively related to employment density, and argues that this is due to the presence of increasing returns deriving from more varied producer services in denser areas, controlling for the effect on wages of good access to an efficient workforce, either locally or as a result of in-commuting. The higher wages in Greater London in particular are seen to be linked to increasing returns and enhanced worker efficiency, although other central city areas show similar influences to a lesser degree, with in-commuting a significant factor.

1. Introduction

This paper tests a model based on the economics underlying the tendency for people and firms to agglomerate together in space and the consequence of this for productivity and for wages. The advantages of agglomeration are clearly in evidence when one considers the fact that the share of the world's population living at higher density (i.e., in cities) has been rising year-on-year and is, according to a UN report published in 1994, expected to exceed 50% by 2005. There is clear empirical evidence (Quigley, 1998) that large densely populated cities are more productive than sparsely populated rural areas or smaller cities. This is not simply because dense areas have a greater level of inputs and thus will naturally produce more. The evidence shows that they produce more than you would expect from the increase in density of economic activity. The increase in productivity with density is accompanied by an increase in wages. Rivera-Batiz and Rivera-Batiz (1998) show that agglomeration externalities help explain geographical wage differentials among the various counties of New York State. In this paper we show that the relationship between the density of activity (in the form of employment density) and wages is discernable at the local authority level in Great Britain, and that this adds further weight to the recent emphasis on increasing returns in the urban and geographical economics literature.

2. The model: theoretical basis

In recent decades the urban and geographical economics literature (Fujita and Thisse, 2000; Huriot and Thisse, 2000; Fujita *et al.*, 1999; Brakman *et al.*, 2001; Abdel-Rahman and Fujita, 1990; Rivera-Batiz, 1988; Ottaviano and Thisse, 2001) has proposed interrelated theories invariably linked to the theory of monopolistic competition developed by Dixit and Stiglitz (1977), explaining why denser areas are likely to be more productive and have higher wages than areas with a lesser concentration of economic activity. The basic mechanism is the existence of scale economies in high-density areas resulting from market interdependence (pecuniary externalities), although we should add to this the increasingly recognised role of positive (technological) externality effects.

To see these, we make use of the simplification of the urban economy commonly used in the urban economics literature, namely that the economy can be divided into two sectors, what we refer to here as a final goods and services sector, which is traded competitively on world markets and has no internal scale economies, and a producer services sector providing inputs to the final goods sector, which is localised, specialised, typified by imperfect competition, and immobile. If we assume a monopolistic competition market structure for the producer service sector, the result is increasing returns that are internal to producer service sector firms. These internal economies translate into productivity gains based on external economies to the final goods sector which are increasing in the density of economic activity, as shown by Ciccone and Hall (1996). This urban economics set up contrasts somewhat with typical 'new economic geography' theory based on competitive agriculture and imperfectly competitive final goods and with explicit consideration of transport costs, although de Vaal and van den Berg (1999) extend new economic geography to include inputs from producer services, recognising the importance of explicit service linkages for agglomeration.

However it is worth noting that agglomeration can be the result without necessarily assuming internal economies in the producer services sector. In this context Anas and Xu (1999) develop a general equilibrium model, with agglomeration an outcome, in which firms' technology is constant returns to scale. Likewise Mori and Nishikimi (2002) emphasise scale economies in transportation, with agglomeration and transport reciprocally reinforced in a circular causation mechanism.[1] Anas *et al.* (1998) also mention that competitive firms with an input-output relationship involving transport costs that co-locate may benefit from external economies.

One of the criticisms of the Dixit and Stiglitz monopolistic competition approach is the absence of strategic interaction, so that firms are myopic, not changing output in response to others' change in price. Neary (2001) for instance points out that in reality firms may create barriers to entry or engage in strategies

[1] They also acknowledge that density economies in transportation are more pronounced when final producers use a wide variety of intermediate inputs.

to stop others eroding their profits, and Anas *et al.* (1998) consider how strategic interaction within spatial oligopoly may itself induce agglomeration.

One reason for the assumption of no strategic interaction within a monopolistic competition framework is that it simplifies the analysis, with the composite level of services a constant in the demand function for each variety of service, in which the price elasticity is constant across all firms (equal to the elasticity of substitution in the CES sub-production function). The assumption is that firms do not change their output in response to competitors' price changes, so there is no feedback to the level of composite services, which remains constant. This leads to the profit maximising price and with free entry the zero profit constant equilibrium level of output per firm. This is constant regardless of the level of total employment per area. Increasing total employment increases the number of firms (varieties) not their individual size. For a given employment density the outcome is an equilibrium number of producer service firms or varieties, an equilibrium level of composite services and therefore an equilibrium level of final goods output (see the Appendix). It is the relationship between final goods output and density that typically will exhibit increasing returns to scale, since higher density means more varieties of imperfectly substitutable producer services and, assuming that final producers prefer variety, output is enhanced by the efficiency gains from more finely divided services. Therefore, as a first approximation, it seems to be reasonable to assume imperfect competition but with a lesser degree of internal scale economies than under oligopoly, a very large number of differentiated services with no strategic interaction, and value attributed to variety.

With increasing density, we might also anticipate increasing negative (congestion) externalities. Congestion provides a brake on how dense an area can become, although as the empirical work in this paper suggests, increasing congestion is apparently more than offset by the productivity gains associated with higher density. Congestion is but one factor that is, on the whole, outside the market and so is unpriced, even though it affects productivity. There are other spill-over effects that are becoming increasing important to the urban economy, relating to knowledge and its enhanced rate of transmission and generation (see for example Audretsch and Feldman, 1996, and Breschi and Lissoni, 2001). The investment made in knowledge production is often not captured completely so there is scope for others to free-ride, gaining benefit from the presence of this external economy. In the paper we therefore emphasise the role played by differential access of areas to workers who are associated with research and development, knowledge generation, production and transmission, as an additional factor influencing productivity and wage rates, and attempt to capture in an indirect way the more elusive technological externalities associated with knowledge flows.

The theory, leading to a function for wage rates, assumes a Cobb-Douglas production function for competitive final goods and services sector output in an area, with inputs consisting of the level of final goods labour efficiency units (M) and the level of composite services (I), and with the level of composite services based on a CES sub-production function for producer services under monopolistic competition. The other input is land (L) but since we are working in densities, $L = 1$. It is possible to show (see the Appendix) that this leads to the reduced form linking the level of output, productivity, wage rates and the density of economic activity. Equation (1) gives the relationship between the level of final goods and services output (Q) and the total number of labour efficiency units N (in both final goods and services and producer services) per unit area

$$Q = (M^{\beta} I^{1-\beta})^{\alpha} L^{1-\alpha} = (M^{\beta} I^{1-\beta})^{\alpha} = \phi N^{\gamma} \tag{1}$$

in which ϕ is a function of other constants and γ is the elasticity where

$$\gamma = \alpha[1 + (1-\beta)(\mu - 1)] \tag{2}$$

In this model increasing returns ($\gamma > 1$) are a result of the increased variety of producer services with the density of activity, subject to diminishing returns due to congestion effects ($\alpha < 1$) (Ciccone and Hall, 1996), and depending on the relevance of services to final production ($\beta < 1$), and on the presence of internal scale economies to producer services ($\mu > 1$).

There are no easily accessible data available for output levels in local authority areas of Great Britain, so instead we work with data on wage rates (see below). The determination of wage rates is assumed to be the equilibrium allocation of labour efficiency units to final production (Q) and to land (L). Let us look first at what is paid to land, and then work out what is paid to labour. Following standard equilibrium theory, we know that output depends on labour efficiency units (N) and on the number of units of land (L), thus

$$Q = [f(N)]^{\alpha} L^{1-\alpha} \tag{3}$$

Differentiating with respect to land gives the equilibrium allocation, hence

$$\begin{aligned} dQ/dL &= f(N)^{\alpha} L^{1-\alpha}(1-\alpha)/L = (1-\alpha)Q/L \\ r &= (1-\alpha)Q/L \\ rL/Q &= 1-\alpha \end{aligned} \tag{4}$$

The derivative is the marginal product of land, so from our competitive equilibrium theory the rent to land is equal to r. This implies that the share of final product being paid to land is the rent per unit of land r times the number of units of land (L) divided by final product (Q), which equals $1 - \alpha$.

Turning next to labour, since there are only two factors of production, land and labour, it must then be the case that the share of Q going to labour efficiency units of both types (N) must be the remainder, in other words α. As with the example of land, this is equal to the wage rate per labour efficiency unit times the number of labour efficiency units divided by final product Q, in other words

$$wN/Q = \alpha \tag{5}$$

In order to pursue our analysis of the determination of wages, we re-express this as

$$\ln(w) = \ln(Q) + \ln(\alpha) - \ln(N) \tag{6}$$

and substitute for Q and for labour efficiency units $N = EA$ in which E is the total employment level per square km and A is each area's level of efficiency, hence

$$\ln(w) = \ln(\phi) + \gamma \ln(AE) + \ln(\alpha) - \ln(AE) \tag{7}$$

It then follows that

$$\ln(w) = k_1 + (\gamma - 1)\ln(E) + (\gamma - 1)\ln(A) \tag{8}$$

in which k_1 denotes a constant.

The next step is to make some assumptions regarding the determinants of A. It seems appropriate to assume, given that we are analysing small area data within the UK, that the key determinant of the variation in efficiency level among areas is attributable to the differences between workers in their ability to make use of the technology that is available. We therefore assume as a first approximation that technology is homogeneous across the areas but there exist differences between areas in terms of the ability to make productive use of that technology. In eq. (10), the natural log of an area's level of efficiency is a linear function of the level of educational attainment of residents of the area as measured by the percentage of the pupils with higher-level educational qualifications (H). The rationale here is the widely recognised link between labour efficiency and schooling. A second indicator of local area efficiency is the technical 'workplace oriented' knowledge (T) of the workforce. This is approximated by the relative concentration of employees in the computing and research and development sectors (see later for a more precise definition of these variables).

We also recognise that workers are mobile, so that labour efficiency within an area is also a function of the efficiency level in other areas from which workers commute (the wage data are based on employer surveys and therefore relate to the place of work not the place of residence). The spillover of efficiency levels between areas is represented by the term $W\ln(A)$ which represents the matrix product of the so-called W matrix and $\ln(A)$, where the definition of W is

$$\begin{aligned} &W_{ij} = \exp(-\delta_i d_{ij}) \qquad i \neq j \\ &W_{ij} = 0 \qquad i = j \\ &W_{ij} = 0 \qquad d_{ij} > 100km \end{aligned} \tag{9}$$

Equation (9) shows that the value allotted to cell (i, j) of the W matrix is a function of the (straight line) distance (d_{ij}) between areas and an exponent δ_i that reflects the area-specific distance decay. The choice of exponent δ_i is based on empirical

Table 1 Commuting distances in Great Britain

Distance	<2 km	2 to 4	5 to 9	10 to 19	20 to 29	30 to 39	⩾ 40 km	
%	26.84	25.21	20.83	15.81	5.00	2.05	4.26	100

comparisons with observed census data on travel to work patterns.[2] Table 1 shows the overall proportion of workers living in Great Britain travelling various distances to work.[3] Given observed travel percentages comparable to Table 1 for each area, the exponent δ_i for each area was chosen by iterating the function $\exp(-\delta_i d_{ij})$ through a range of values to obtain the value giving the closest fit to each area's commuting data.[4] The idea behind this approach is that, for any given area, the relative weights of neighbouring areas should approximate to the ratio of the number of commuters travelling in from each. In 1991 4.26% of workers commuted further than 40 km, so we allot weights up to distances of 100 km to accommodate long-distance commuting, with areas further than 100 km given zero weight. Two extreme cases are observed. One is the case of London local authorities, for which the best-fit exponents are well below the average for Great Britain, reflecting the fact that there is only a shallow distance decay for commuting in London. For example, the data show that 22% of workers in the City of London local authority travelled at least 40 km to work.[5] The other extreme case is that of areas that are so remote that none of the surrounding areas carries any weight using this formula. However, in reality this is not the case, because often there are special transport arrangements in such areas, for instance the data indicate that 11.9% of workers in Scilly travel more than 40 km, obviously coming from the mainland. In order to offset the effect of excessive remoteness, in these cases the distances to places within 100 km where scaled by a factor of 2/3.[6]

The contribution to area i's efficiency level from in-commuting is given by row i of vector $W\ln(A)$ which contains the sum of the weighted efficiency levels of all other areas within 100 km. There are also other variables, apart from the constant, which we could no doubt introduce to more precisely capture the determinants of efficiency levels, but a complete enumeration of all relevant variables is impracticable. For the moment assume that these excluded variables behave as

[2] 1991 Census of Population – Special Workplace Statistics, available from NOMIS.

[3] Total employees and self-employed with a workplace coded.

[4] Minimum of the sum of the squared deviations of the observed proportions in each distance band up to 40 km and the proportions of the sum of the function $\exp(-\delta_i\, d_{ij})$ calculated using the upper limit of each distance band.

[5] This suggests that there may be endogenous interaction involving agglomeration and the transport infrastructure (see Mori and Nishikimi, 2002), and raises the question of how a dynamic version of the model discussed here might be developed.

[6] Isles of Scilly, Eilean Siar, Highland, Orkney Islands, Shetland Islands.

random shocks (ξ). Note that by making $\ln(A)$ depend on $W\ln(A)$ and not simply WH and WT, we capture the totality of the effects influencing the efficiency level, including those represented by the random shocks. This specification has rather important implications for the estimation method we of necessity adopt in the paper, since it creates an endogenous spatial lag and necessitates therefore the methodology of spatial econometrics. We highlight the presence of the spatial lag via the coefficient ρ.

Combining the variables assumed to affect an area's efficiency level we obtain

$$\begin{gathered} \ln(A) = b_0 + b_1 H + b_2 T + \rho W \ln(A) + \xi \\ \xi = N(0, \sigma^2) \end{gathered} \tag{10}$$

Since we cannot substitute for $\ln(A)$ in eq. (8) because we do not know $W\ln(A)$, we need to find $W\ln(A)$ in terms of known variables. Re-arranging eq. (8) and multiplying by W, we can see that the following holds true

$$W \ln(A) = W \frac{-k_1}{\gamma - 1} + \frac{1}{\gamma - 1} W \ln(w) - W \ln(E) \tag{11}$$

It now follows that, adding an error term (ω) allowing for measurement error in the wage variable

$$\begin{aligned} \ln(w) = k_1 + (\gamma - 1)\ln(E) + (\gamma - 1)\Bigg[& b_0 + b_1 H + b_2 T \\ & + \rho\left(W \frac{-k_1}{\gamma - 1} + \frac{1}{\gamma - 1} W \ln(w) - W \ln(E) \right) + \xi \Bigg] + \omega \end{aligned} \tag{12}$$

Hence

$$\begin{gathered} \ln(w) = k_2 + \rho W \ln(w) - \rho W k_1 \\ + (\gamma - 1)(\ln(E) - \rho W \ln(E)) + a_1 H + a_2 T + \upsilon \\ \upsilon \sim N(0, \tau^2) \end{gathered} \tag{13}$$

in which k_2 is a constant and Wk_1 is a variable that depends on the unknown values of ϕ and α, although this can be ignored without effect.

One alternative to this approach would be to work with the standardised W matrix so that as a consequence Wk_1, the matrix product of W and the constant vector $-k_1$, becomes a constant. However we prefer to work with the W matrix based on absolute distances rather than rely on the relative distances created by standardisation.

3. Data

The data on wage rates (w) by unitary authority and local authority districts in Great Britain are measures of gross weekly pay (all occupations, all persons) taken from the Office for National Statistics' New Earnings Survey.[7] The Office for National Statistics carries out the New Earnings Survey by taking a 1% sample of employees who are members of Pay-As-You-Earn (PAYE) income tax schemes. The Survey therefore is concerned with evaluating the earnings of employees in employment and covers all the categories of employee employed in different kinds of businesses and businesses of all sizes. Employees who appear in the PAYE taxation system in February of each year form the sampling frame from which the 1% sample is drawn at random. However it is the employer, not the employee who completes the postal questionnaire, and the sample is supplemented by data provided by large employers, using extracts from their payroll systems. While there is no grossing, weighting, or imputation undertaken for non-response or sample frame deficiencies, there are checks on accuracy, for instance if an employee is working in a different geographical area to the previous year. While the response rate is nearly 70%, which is high by postal survey standards, and the overall numbers involved are large, when broken down by area there will be considerable sampling error, although the distinct aggregate regional patterns which are a familiar feature of the UK economy are also present in the data at the unitary authority and local authority district level.[8] In 1999, approximately 233,000 questionnaires were distributed, of which approximately 160,000 returns were finally used to compile the published data, an average of nearly 400 per area (although the actual number of responses will vary considerably between areas). The range of variation of the wage data is apparent from Fig. 1, with gross weekly pay ranging from £814.95 for the City of London to £212.52 for Alnwick in the North East of England.

The 'employment density' (E) for each year is based on the annual business enquiry employee analysis, also carried out by the Office of National Statistics and available on the NOMIS database. For each area, the employment density was calculated by dividing the total employee level by its area in square km. Data on the geographic area of each unitary authority and local authority district was provided by the Office of National Statistics. Figure 1 shows the range of variation of this variable.

The schooling data (H) are the proportions of pupils in each area achieving two or more A level (or three or more Scottish higher) qualifications in 1991. It is therefore a broad measure of the academic skills present on the local population. Figure 2 shows that the mean is about 20% but some areas have as few as 5% of

[7] Available on the NOMIS website (the Office for National Statistics' on-line labour market statistics database).

[8] There are no data for Scilly isles, so the data for the nearest mainland area of Penwith have been used in this case.

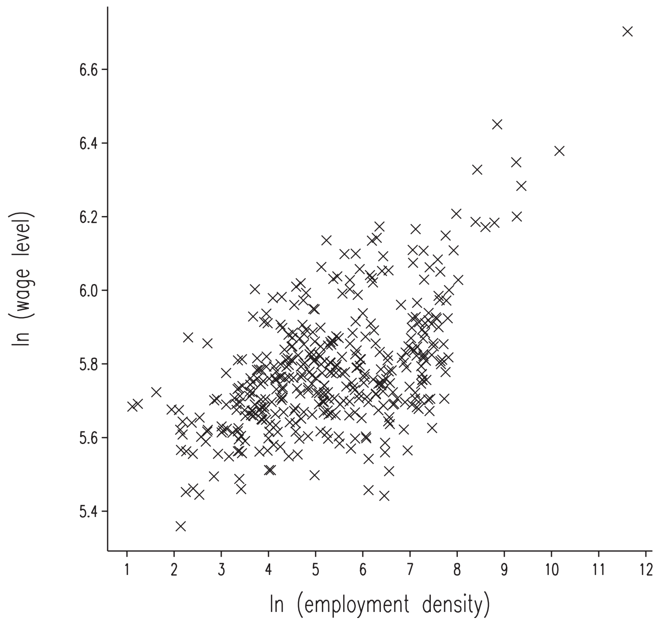

Fig. 1. The relationship between the natural log of the wage level and employment density in 408 unitary authority and local authority districts of Great Britain

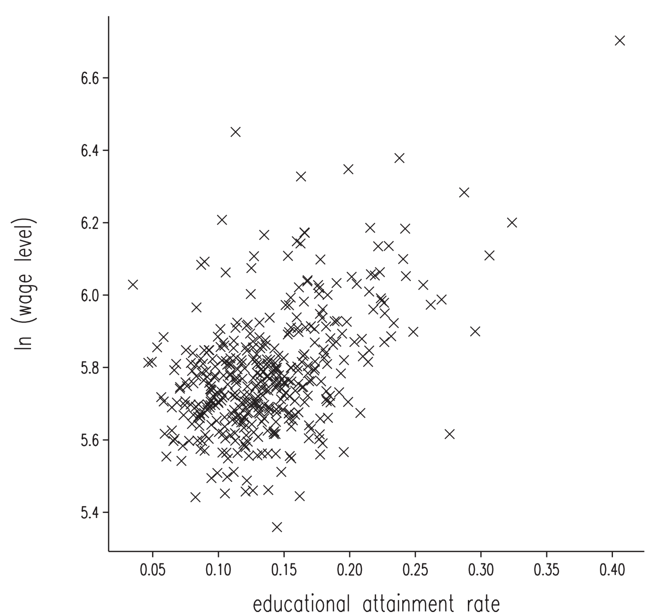

Fig. 2. The relationship between the natural log of the wage level and educational attainment rates in 408 unitary authority and local authority districts of Great Britain

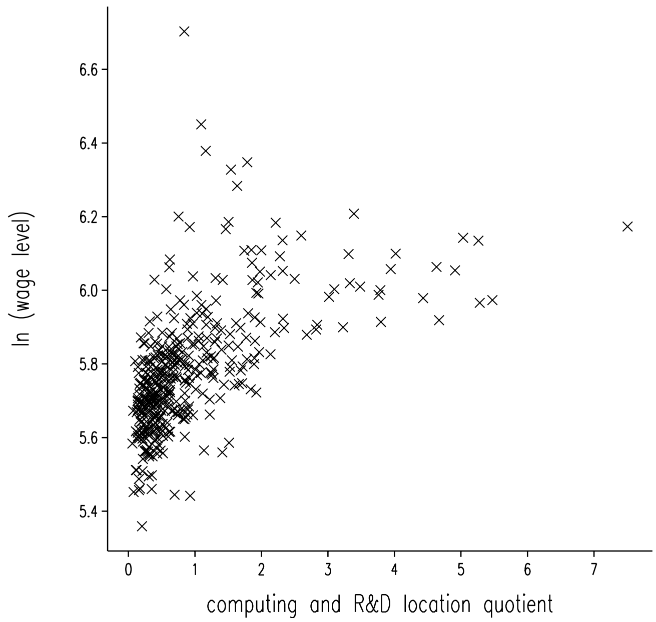

Fig. 3. The relationship between the natural log of the wage level and the relative concentration of employment in Computing and Research and Development in 408 unitary authority and local authority districts of Great Britain

pupils achieving this level, while a few have more than 30%. The 'technical knowledge' variable (T) is the location quotient for each area giving the workforce specialisation in computing and related activities (1992 SIC 72) and in research and development (1992 SIC 73), calculated from data taken from the annual business enquiry employee analysis (available through NOMIS).[9] This therefore measures the relative concentration by area of employees with work-related skills in hardware consultancy, software consultancy and supply, data processing, data base activities, computer and office machinery maintenance and repair, and in other unspecified computer related activities. In addition it includes workers involved in research in the natural sciences and engineering, and in the social sciences and humanities. Figure 3 shows the distribution of the location quotients, with the majority of areas having a share that is slightly below the national share, while a few specialised localities have share that are as much as five or seven times the national share.

4. Preliminary analysis

The specification given in eq. (13) indicates that we should see increasing wage levels with increasing density of employment. Figure 1 is a plot of the observed

[9]The location quotient is the share of local employment in these sectors divided by national share.

natural log of the values of these two variables for the 408 areas for the year 2000. The issue here is whether this apparently significant linear relationship holds, controlling for the other variables, since that would indicate that there are increasing returns to the density of activity. We address this issue in the multivariate context below. Figure 2 suggests a possible link between the level of schooling (H) and the log of the wage level. In this case the significance of the relationship is more questionable although there does appear to be a positive relationship as anticipated by the foregoing theory. Figure 3 plots the wage level against the second component of labour efficiency (T), the relative concentration of workers in computing and research and development. It appears that there are a number of outliers and that the relationship is non-linear, however there is a broadly rising level of wages with increasing values of T indicating the possible significance of this variable. Finally Fig. 4 shows the empirical relationship between the log level of wages and $W\ln(w)$, the sum of wage levels in surrounding areas weighted according to the estimated commuting inflow. The apparent significance of this relationship would support the specification given as eq. (10) since both this equation and eq. (13) have the coefficient ρ in common. Equation (10) hypotheses that the efficiency level of an area ($\ln(A)$) is partly a function of that of surrounding areas ($W\ln(A)$) due to the flow of workers embodying knowledge and skills across area boundaries.

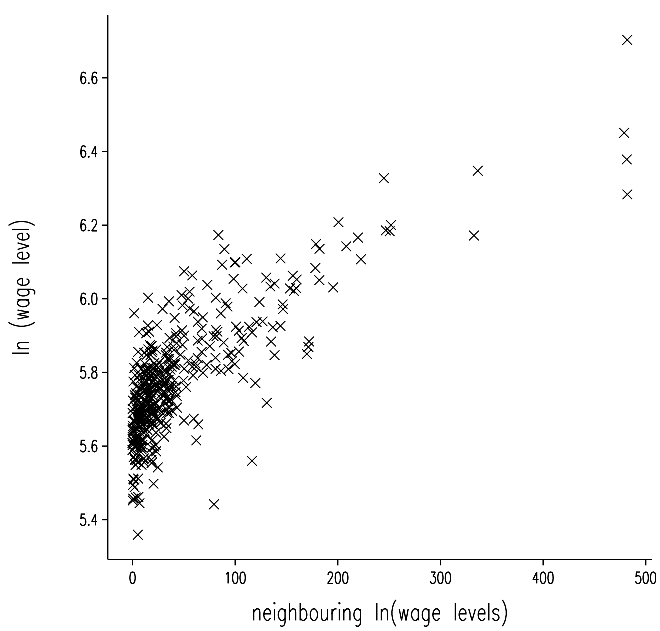

Fig. 4. The relationship between the natural log of the wage level and wages levels in 'neighbouring' areas for 408 unitary authority and local authority districts of Great Britain

5. Econometric results

The first round of estimation is based on the specification

$$\ln(w) = k_2 + \rho W \ln(w) + (\gamma - 1)(\ln(E) - \rho W \ln(E)) + a_1 H + a_2 T + \upsilon \qquad (14)$$

which simply omits Wk_1. This provides the estimates given in Table 2. The model defined by eq. (14) was fitted to two cross-sectional series for the years 1999 and 2000. Estimation is via two stage least squares, since it is well known (Anselin, 1988) that the presence of a so-called endogenous lag on the right hand side ($W\ln(w)$) means that OLS estimation is inconsistent.[10] In addition, the employment density of each area (E) is also likely to be an endogenous variable, since it will be a response to the wage rate as well as a determinant of it. On the other hand, we assume that the variables H and T are exogenous, since they pre-date the year of analysis. This leads to a number of issues relating to two stage least squares estimation, such as the choice of instruments, the possibility of an unstable model, the diagnostics, and way in which the restriction on the coefficient for $W\ln(E)$ evident in eq. (14), so that it is equal to $((\gamma - 1)\rho)$, is satisfied.

With regard to the choice of instruments, the method used is based on the three group method (described in Kennedy, 1992, and Johnston, 1984) in which the instrumental variable takes values 1, 0 or -1 according to whether E is in the top, middle or bottom third of its ranking, which ranged from 1, the area with

Table 2 Two stage least squares estimates for 1999 and 2000

Parameter	estimate	standard error	t ratio
constant (k_2)	[5.509189]	[0.026742]	[206.016438]
	5.550149	0.026127	212.430592
spillover (ρ)	[0.001310]	[0.000094]	[13.927574]
	0.001377	0.000091	15.177157
increasing returns ($\gamma - 1$)	[0.017602]	[0.003983]	[4.419225]
	0.015825	0.003891	4.067277
schooling (a_1)	[0.302457]	[0.120676]	[2.506365]
	0.271390	0.118022	2.299484
technical knowledge (a_2)	[0.046388]	[0.005241]	[8.850553]
	0.052509	0.005287	9.930750
error variance (τ^2)	[0.008410]		
	0.008006		
R-squared	[0.6981]		
	0.7213		
Correlation	[0.6982]		
	0.7238		

Notes: [ ] encloses the estimates based on 1999 data.

[10]Unreported maximum likelihood estimates were also obtained for the models discussed in this paper. These produce estimates very similar to those reported here.

the lowest employment density, up to 408.[11] We denote this instrument by E_I and assume that no correlation exists between E_I and υ induced by measurement error and by simultaneity. Additional instruments are also available, namely the exogenous variables H and T and the spatial lags WE_I , WH, and WT. [12]

The potentially 'explosive' or non-convergent nature of the model results from the unbounded range of the two stage least squares estimator of the coefficient ρ, which unlike the ML estimator can fall outside the stable range and be close to or equal to a singular point. For example, if we write our model in matrix terms so that

$$\ln(w) = (I - \rho W)^{-1}(Xb + \upsilon) \tag{15}$$

where X is the n by k matrix comprising the unit vector and $k-1$ regressors and b is a k by 1 vector consisting of the coefficients, then the inversion requires a non-singular matrix but $I - \rho W$ becomes singular for instance at the points $\rho = 1/\mathrm{e}_{\max}$ and $\rho = 1/\mathrm{e}_{\min}$ where $\mathrm{e}_{\max} = 24.015$ is the largest positive eigenvalue of W and $\mathrm{e}_{\min} = -0.9791$. Fortunately, the IV estimated ρ lies within these bounds.

The estimation method involves an iterative routine in order to satisfy the restriction that the combined endogenous employment density variable is equal to $(\ln(E) - \rho W\ln(E))$.[13] At the first iteration, with no initial estimate for ρ, we carry out the two stages of two stage least squares arbitrarily assuming that $\rho = 1$. The first stage involves regressing $(\ln(E) - W\ln(E))$ and the endogenous lag $W\ln(w)$ on the instrumental variables (E_I, H, T, WE_I ,WH and WT) to obtain fitted values (the instruments). The second stage uses both instruments in place of the two endogenous regressors to obtain estimates of the coefficients, including an estimate of ρ. The second iteration uses estimated ρ to recalculate the endogenous variable $(\ln(E) - \rho W\ln(E))$. Then two stage least squares is carried out again to obtain revised parameter estimates. The third iteration is based on the estimated ρ from this second iteration and so on until the parameter estimates converge. Table 2 summarises the results obtained by this process.

The estimates in Table 2 are consistent with the theory outlined above.[14] The fact that $\gamma - 1$ is significantly greater than 0 provides evidence to support the idea that there are increasing returns to density of employment. There is also a significant boost to the level of wages which is attributed to the heightened efficiency of workers in areas with high levels of educational attainment and technical knowl-

[11]This method is described in the context of variables subject to measurement error, but is intended here to have the same effect of eliminating correlation between the instrument and the error term.

[12]See Kelejian and Robinson (1993), Kelejian and Prucha (1998) for a discussion of the efficacy of the use of low order spatial lags. While the use of spatial lags is seen as an effective way to generate instruments, these authors warn against including high order spatial lags to avoid linear dependence.

[13]Written in the GENSTAT programming language.

[14]The 1999 estimates are based on 1999 wage and employment levels, and on 1998 data for the proxy for the variable T. The 2000 estimates use 2000 wage and employment data and 1999 for the T proxy. In both cases 1990 data are used for H and the W matrix.

edge, and the sign and significance attributed to ρ supports the theory that an area's efficiency level also depends on the efficiency level of 'neighbouring' areas.

While the overall goodness-of-fit as given by R^2 and the square of the correlation between the observed and fitted values are reasonably satisfactory, we know from theory of at least one omitted variable (Wk_1) and the diagnostic indicators also point to misspecification. We first test the residuals produced by the two stage least squares estimates for spatial autocorrelation, using the appropriate statistic developed by Anselin and Kelejian (1997) together with Moran's I applied to the IV residuals which provides an informal measure and which is also a measure of error misspecification in general.[15] In fact both statistics point to fairly well behaved residuals for the 2000 data.[16] However, when we examine the relationship between the raw residuals and the observed log of wages levels for the 2000 data, we obtain a significant positive correlation ($r = 0.5279$. For 1999 the comparable figure is 0.5495). There appear to be additional factors, omitted from the model, which boost the wage rate in the higher wage areas and depress it in lower wage areas.

One such factor could be spatial inhomogeneity in sectoral composition which, so far as it is not picked up by H and T, may be an omitted variable leading to bias in our estimates. The most obvious example of this is the concentration of highly paid jobs in specialised services particular to the City of London.

A second potential source of misspecification is the exclusion of technological externalities (other than the impact of un-priced congestion), in this case involving knowledge spillovers (i.e. Jacobs externalities and MAR externalities).[17,18] However, while according to Glaeser (1999), 'Urban economics needs to specialize in non-market interactions' and 'the flow of ideas and values that occurs through face-to-face interaction' may be the most interesting feature of the city, it is unclear as to how one could incorporate these more ephemeral influences into an econometric model.[19] As Krugman (1991) maintains, knowledge spillovers between firms are 'invisible' and 'leave no paper trail by which they can be measured or tracked'.

[15]The existence of omitted (spatially autocorrelated) regressors could be manifest as autocorrelated residuals.

[16]The appropriate statistic with an endogenous lag and endogenous regressors gives a standardised value equal to 2.659 (1999) and 1.002 (2000), the latter not being significant when referred to the upper 5% of the $N(0, 1)$ distribution. The informal test based on testing the two stage least squares residuals using Moran's I for regression residuals (with randomisation moments) gives a value of 5.128 (1999) and 2.187 (2000).

[17]This is assuming that the variables H and T are not also picking up knowledge spillover effects.

[18]Jacobs or urbanization externalities (after Jacobs, 1969) are external to the firm and sector but internal to the city, Localization or Marshall-Arrow-Romer (MAR) externalities are external to the firm but internal to the sector.

[19]Most economic geography research focuses on large geographic areas, such as nations and states. In contrast Wallsten (2001) and Duranton and Overman (2002) use firm-level dataset to explore clustering, agglomeration and spillovers at the firm level. The level of resolution in this paper is small areas at the sub-county level, which is effective for city specific, rather than firm specific, variables such as intra-urban variations in density and congestion.

On the contrary, Quigley (1998) argues that while we cannot observe knowledge as it spills out 'among the buildings and streets of a city', some of this spillout does leave a paper trail. Cameron (1996) for instance observes that researchers typically use some proxy for the flow of spillovers; such as input-output tables, patent concordances, innovation concordances, and proximity analysis. For example Jaffe *et al.* (1993) trace knowledge spillovers using patent citations.

However, it is questionable whether such a specific proxy variable approach would be successful in capturing the totality of omitted external economies, and in this regard we concur with the views of Gordon and McCann (2000), who argue that we can only truly observe the net realized effects of diverse simultaneous externality mechanisms, rather than individual sources. In what follows, we assume that estimated errors from a model that excludes technological externalities incorporate their net realised effects.

We therefore seek to improve our estimation of the 2000 wage rates using the estimated 1999 errors as a proxy for the effects we assume are omitted and which may be a cause of estimation bias, although it turns out that introducing this proxy makes little difference to our results. The assumption is that the net outcome of omitting sectoral composition and externalities is given by estimated 1999 errors that are represented by r_{-1} in eq. (16)

$$\ln(w) = k_2 + \rho W \ln(w) + (\gamma - 1)(\ln(E) - \rho W \ln(E)) + a_1 H + a_2 T + a_3 r_{-1} + \upsilon \quad (16)$$

where the –1 signifies that these are from the model fitted to data for the previous year and the free coefficient a_3 reflects the assumption that r_{-1} represents more than Wk_1.

The 1999 estimated errors undoubtedly contain measurement errors, since they apply to 1999 and not 2000, and for this reason alone the estimation process needs to eliminate inconsistency due to any relationship between this variable and the model errors. The method employed is again two stage least squares, so for the purposes of iterative estimation we now have three variables for which instruments are required, namely r_{-1}, $\ln(E) - \rho W\ln(E)$, and $W\ln(w)$. However we also have two additional instrumental variables, $(r_{-1})_I$ which equals –1, 0 and 1 depending on the r_{-1} value, and $W(r_{-1})_I$ which is the result of the matrix multiplication of $(r_{-1})_I$ and the W matrix. The results given in Table 3 are the outcome of a sequence of iterations identical to that used to obtain the estimates in Table 2, but in this case with the additional variable r_{-1} and using the instruments $(r_{-1})_I$, $W(r_{-1})_I$, E_I, H, T, WE_I, WH, and WT.

Table 3 shows that while the inclusion of r_{-1} soaks up a significant proportion of the previously unexplained variation in wage rates across areas, its presence does not affect the interpretation one would place on the effects of the substantive variables in the model, except by enhancing their statistical significance.

The correlation between the residuals from the Table 3 model and the observed log wage rates is considerably reduced but not completely eliminated by this procedure ($r = 0.3159$). The residual autocorrelation statistics are -0.9538 and -0.9722 for Moran's I for the two stage least squares residuals, neither of which is significant.

Table 3 Final model estimates

Parameter	estimate	standard error	t ratio
constant (k_2)	5.546059	0.016352	339.171359
spillover (ρ)	0.001373	0.000057	24.123611
increasing returns ($\gamma - 1$)	0.016446	0.002435	6.754561
schooling (a_1)	0.292881	0.073886	3.963973
technical knowledge (a_2)	0.050397	0.003311	15.219963
'omitted' variables (a_3)	0.776197	0.035484	21.874380
error variance (τ^2)	0.003138		
R-squared	0.9004		
Correlation	0.8921		

6. Interpretation of the model

The models summarised by Tables 2 and 3 point to statistically significant effects due to increasing returns to employment density ($\gamma - 1$), and a significant relationship between wage rates and labour efficiency levels within areas as reflected by educational attainment rates H (a_1) and the concentration of employment in computing and research and development T (a_2). We also see that wages are significantly related to wage rates in other 'nearby' areas (ρ), which is interpreted as an effect operating via the dependence of an area's labour efficiency on efficiency levels of other areas and which is caused by commuting. Finally, observe that a_3 is very significantly different from zero, suggesting that there is a range of other external economies and omitted variables that, while unknown in detail, are at least embodied in our model in an indirect way and this is justified by the fact that they do have a significant effect.

However, while the Table 3 estimates provide empirical support for the underlying theoretical model, the parameter estimates are simply global measures that provide no information about the differentiated quantitative importance of their various effects in different areas. In order to highlight the impact of differentiated access to efficient labour and productivity enhancing producer service variety, the model is used to simulate wage rates that would occur assuming that a given variable had no effect. The simulated wage levels are then compared to the actual wage levels in each area in order to illustrate the local importance of the nullified variable.

The effect of each variable is nullified by setting the variable's values equal to zero across areas, which is equivalent to setting its coefficient to zero.[20] Equating a

[20] If there were no spatial effect (i.e. $\rho = 0$) then the equalized level would be arbitrary in terms of the comparative impact across areas. The impact would be the same regardless, in the sense that the ratio of simulated wage level w_s to wage level w for area j, $r_j = w_{sj}/w_j$, increases if we increase the equalized level, but the areas will maintain the same pecking order, and the ratio of their ratios (r_j/r_k, $k = 1 \ldots 408$) is constant irrespective of the assumed equalized level of the nullified variable. It is possible to show that this is not true when $\rho \neq 0$.

Table 4 The impact of increasing returns

Rank	area	100*ratio
1	City of London	82.6151
2	Westminster, City of	84.6006
3	Camden	85.7320
4	Kensington and Chelsea	85.8681
5	Islington	85.8865
6	Tower Hamlets	86.4582
7	Hammersmith and Fulham	86.5408
8	Southwark	86.8118
9	Hackney	87.0613
10	Lambeth	87.1125
11	Wandsworth	87.6395
12	Slough	87.6985
13	Watford	87.7810
14	Reading	87.9213
15	Portsmouth	87.9396
16	Manchester	87.9598
17	Brent	88.0029
18	Hounslow	88.0254
19	Nottingham	88.0280
20	Leicester City	88.0662

given variable's coefficient to zero, while allowing the remaining variables to have their estimated effect, thus isolates the relative impact by area of the variable in question. The outcome is the set of wage level ratios, each below the value 1, given in Tables 4 to 7.

When we set to zero the coefficient $\gamma - 1$ on the employment density variable, then this is equivalent to switching off the effect of increasing returns since of course the simulation then assumes that $\gamma = \alpha[1 + (1 - \beta)(\mu - 1)] = 1$. It seems reasonable to assume that increasing returns could turn into diminishing returns as a result of additional congestion, and the simulation with $\gamma = 1$ relates to that specific point at which α has attained a value that exactly offsets internal increasing returns in the producer services sector ($\mu > 1$, $\beta < 1$) spilling over as an externality to production in the rest of the economy.[21] Table 4 shows the 20 areas experiencing the largest proportionate decrease in wage level as a result of nullifying the effect of increasing returns.[22] It is apparent that parts of inner London are prominent in this top 20 list, with the simulated wage level almost 18% below the actual level in the City of London. These are the areas that evidently gain most from the benefits to productivity and wage levels as a result of the density of economic activity and diversity of producer services, and the areas that apparently have most to lose from increasing congestion.

[21] However, γ falling to 1 is also consistent with no enhanced congestion effect but changes to the level of internal scale economies or the relevance of the producer service sector.

[22] $100^*w_s/w$.

Table 5 The impact of schooling

Rank	area	100*ratio
1	City of London	88.2994
2	Kensington and Chelsea	90.6724
3	Richmond-upon-Thames	91.2458
4	Camden	91.4148
5	Cambridge	91.6582
6	East Renfrewshire	92.2251
7	St Albans	92.2973
8	Oxford	92.5599
9	Wandsworth	92.5915
10	Westminster, City of	92.7416
11	Hammersmith and Fulham	92.8605
12	Chiltern	92.8887
13	Elmbridge	92.9446
14	Wokingham	93.0716
15	Kingston-upon-Thames	93.2223
16	Mole Valley	93.2832
17	Waverley	93.3665
18	Haringey	93.4731
19	South Cambridgeshire	93.4960
20	Guildford	93.5100

Table 5 shows the effect of nullifying the effect of schooling differentials (H). Overall the impact on wages is less that the impact of returns to scale, but the London and South East region again stands out as benefiting most from schooling.

Table 6 shows the areas most affected by eliminating the contribution to worker efficiency of a concentration of computing and research and development employment (T). Here we see that there are quite important changes in areas in the Thames valley and the region north of London towards Cambridge.

Table 7 shows the impact of eliminating spillover effects across areas ($\rho = 0$). It clearly shows the extent to which high inner London wage levels depend on the influx of efficient workers from throughout the South East region.

Table 8 summarises the impact of the variables for some of the principal city centres of the UK. The key features of this table are the comparative importance of scale economies for most cities, and the relative unimportance of local schooling and technical knowledge compared to the significance of commuting, although Edinburgh stands out as an exception in this regard.

7. Conclusions

This paper provides new evidence that there are increasing returns to the density of economic activity in urban areas, and thus adds to the growing body of empirical work supporting one of the central tenets of recent urban and geographical

Table 6 The impact of computing and research and development

Rank	area	100*ratio
1	Bracknell Forest	68.3331
2	Oxford	75.7885
3	Surrey Heath	76.4917
4	Harlow	76.5403
5	Runnymede	77.1986
6	Woking	77.8567
7	Stevenage	78.8999
8	Hart	79.0386
9	South Cambridgeshire	79.8449
10	Wokingham	81.4332
11	Windsor and Maidenhead	81.6728
12	Reading	82.4733
13	Stratford-on-Avon	82.5279
14	St Albans	82.5578
15	Vale of White Horse	83.7881
16	Slough	83.8593
17	Wycombe	84.4026
18	West Berkshire	84.4257
19	Cambridge	84.9004
20	Dacorum	85.3824

Table 7 The impact of commuting

Rank	area	100*ratio
1	Camden	51.5833
2	City of London	51.5938
3	Westminster, City of	51.6259
4	Tower Hamlets	51.8057
5	Islington	63.0129
6	Southwark	63.3369
7	Kensington and Chelsea	70.7709
8	Hammersmith and Fulham	70.8908
9	Lambeth	71.2597
10	Hackney	71.4468
11	Hillingdon	73.6657
12	Spelthorne	73.9689
13	Runnymede	75.1281
14	Slough	75.9193
15	South Buckinghamshire	76.4611
16	Mole Valley	77.8683
17	Ealing	77.8903
18	Hounslow	78.2388
19	Newham	78.3037
20	Thurrock	78.9617

Table 8 Comparisons of major city centres

	γ Increasing returns		a_1 schooling		a_2 technical knowledge		ρ spillover	
	rank	ratio	rank	ratio	rank	ratio	rank	ratio
City of London	1	82.62	1	88.30	118	94.90	2	51.59
Manchester	16	87.96	264	96.61	174	96.52	45	85.55
Birmingham	33	88.41	310	96.98	156	96.07	51	86.74
Liverpool	35	88.45	360	97.44	238	97.53	92	91.92
Leeds	100	89.81	224	96.33	162	96.26	127	94.77
Newcastle	49	88.71	174	95.97	211	97.10	121	94.50
Bristol	23	88.17	123	95.52	114	94.75	190	96.73
Glasgow	24	88.18	289	96.80	233	97.46	133	95.04
Edinburgh	77	89.09	22	93.57	103	94.21	237	97.57

economics theory. The analysis is based on wage rate variations among the 408 unitary authority and local authority districts of Great Britain, taking account of other factors such as the efficiency of each area's labour force and the presence of technological externalities. The study shows that as the density of workers in an area increases, there is a more than proportionate increase in wage levels, and (since the coefficient γ is the same) this implies that there is a more than proportionate increase in the level of output of final goods and services. The effect seems to have added about 18% to the level of wages in the City of London, and although the impact is less, it is also seen to be the dominant factor enhancing the level of earnings in other major city centres.

One of the important findings in the paper is the significant effect of commuting on wages and productivity in central cities. This brings into focus the interpretation that without commuting, wages and productivity in central areas would be much lower, and highlights the comparatively low income levels of many inner city residents. Perhaps policy should focus even more on raising skills levels in inner areas, possibly by training unskilled workers and by promoting residential development that would appeal to workers with appropriate skills, so as to satisfy a larger portion of the demand that evidently exists and reduce the need for in-commuting. This raises the question of the long term sustainability of highly concentrated urban systems dependent on in-commuting. While there is a strong indication from the available data that increased density is rewarded by increased productivity, if in the longer run this encourages even more agglomeration and cities reach even greater densities than currently observed, congestion may eventually neutralize or even overcome increasing returns. For example, over a wider range of data than currently observable, the apparently log-linear relation between employment density and wages (Fig. 2) and hence production may turn out to be quadratic. It follows that with so much capital sunk into the built environment and transport infrastructure of monocentric city systems, inefficiency in the long term may be inescapable, since the costs of redesigning cities to avoid congestion effects may be so large that we prefer to accept the sub-optimality implied by the status quo.

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Appendix

Assume a Cobb-Douglas production function for the output Q of the competitive final goods and services sector

$$Q = (M^{\beta} I^{1-\beta})^{\alpha}$$

with inputs sector-specific labour efficiency units (M) and composite producer services (I). Assume that at equilibrium each producer service has output equal to $i(t)$, which is constant irrespective of the size N of an area's economy, and there are x firms, depending on the total effective labour N, so that from the CES production function we obtain the simplification

$$I = x^{\mu} i(t)$$

in which μ is a measure of internal returns to scale in the producer service firm at equilibrium. It also determines the constant price elasticity of demand (*ped*), since from the constant demand function

$$i(t) = kp^{-\mu/(\mu-1)}$$

$$\mathrm{d}i(t)/\mathrm{d}p = -\frac{kp^{-\mu/(\mu-1)}\mu}{(\mu-1)p}$$

$$ped = -\frac{\mathrm{d}i(t)/\mathrm{d}p}{i(t)/p} = \frac{i(t)\mu}{(\mu-1)p}\frac{p}{i(t)} = \frac{\mu}{\mu-1}$$

And it also determines the constant elasticity of substitution, since

$$-\frac{\mathrm{d}i(t)}{\mathrm{d}p_t}\frac{p_t}{i(t)} = \frac{\mu}{\mu-1}$$

Given the simple function I, it then is possible to substitute so that

$$Q = (M^{\beta}(x^{\mu} i(t))^{1-\beta})^{\alpha}$$

and therefore

$$Q = M^{\beta\alpha} x^{\alpha(\mu-\mu\beta)} i(t)^{\alpha(1-\beta)}$$

The number of producer service firms x is equal to the producer service effective labour divided by the effective labour per firm, so that

$$x = \frac{(1-\beta)N}{ai(t)+s}$$

where $(1-\beta)$ is the share of effective labour in the producer service sector under competitive equilibrium in the labour market, a is the marginal labour requirement and s is the fixed labour requirement (hence internal increasing returns exist for producer service firms). It then follows that

$$Q = N^{\alpha(\beta+\mu-\mu\beta)}\beta^{\alpha\beta}(ai(t)+s)^{\alpha\mu(\beta-1)} i(t)^{\alpha(1-\beta)}(1-\beta)^{-\alpha\mu(\beta-1)}$$

and simplification gives

$$Q = \phi N^{\alpha(1+(1-\beta)(\mu-1))} = \phi N^{\gamma}$$

Also, since it is easy to show that

$$\ln(Q/N) = \frac{\ln(\phi)}{\gamma} + \left[\frac{\gamma-1}{\gamma}\right]\ln(Q)$$

and

$$\ln(Q/M) = \frac{\ln(\phi)}{\gamma} + \left[\frac{\gamma - 1}{\gamma}\right] \ln(Q) - \ln(\beta)$$

Assume that labour efficiency units are unskilled labour in the final goods sector (F) multiplied by the level of efficiency (A)

$$M = FA$$

then

$$\ln(Q/F) = \frac{\ln(\phi)}{\gamma} + \left[\frac{\gamma - 1}{\gamma}\right] \ln(Q) - \ln(\beta) + \ln(A)$$

So the log of the level of labour productivity in the final goods and services sector is a linear function of the log of the level of final output. It is worth noting that, when final goods are manufactures, this is an equivalent reduced form to the static Verdoorn law discussed by Fingleton and McCombie (1998), although in this paper the coefficient on Q, which if greater than zero indicates increasing returns, is a function of the fundamental parameters α, β, and μ.