EC408 Topics in Applied Econometrics

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Limitations of graphical methods

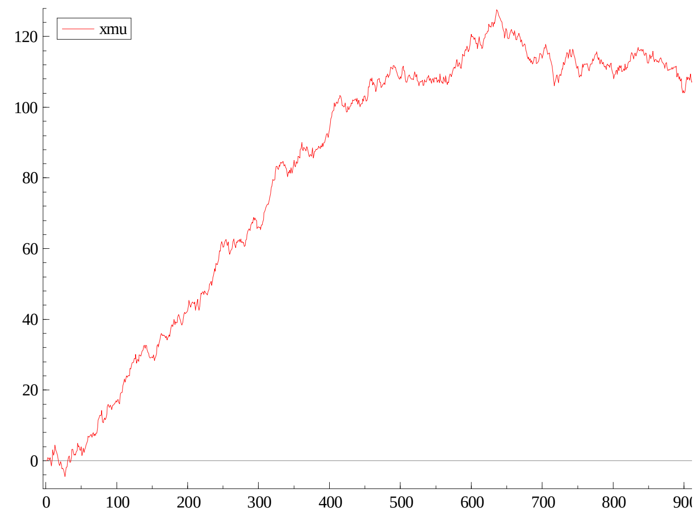

Fig. 3 A Stochastic trend

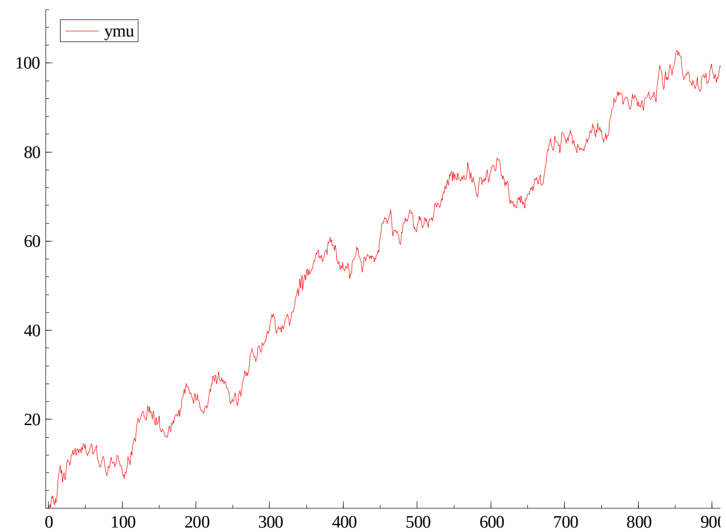

Fig 4 An apparent TSP which is actually a Stochastic trend

Figure 3 gives good visual evidence of a stochastic trend,
but Figure 4 is less clear cut , although it also is an I(1) variable

For the Figure 4 variable a more accurate test is need because visual inspection may be misleading. A good test would show that the variable is also I(1), even though visually it appears to be a TSP.

Limitations of graphical methods

Given an I(1) variable, say Y_t, then by definition $\Delta Y = Y_t - Y_{t-1} = u_t$ ~I(0), in which case

$$Y_t = \rho Y_{t-1} + u_t \qquad (21)$$

$$\rho = 1$$

If $\rho = 1$ then Y_t~I(1)

if $\rho < 1$, then Y_t~I(0)

Y_t has a constant mean and variance,

the correlation between pair of values Y_t and Y_{t+k}

the same for any t and k.

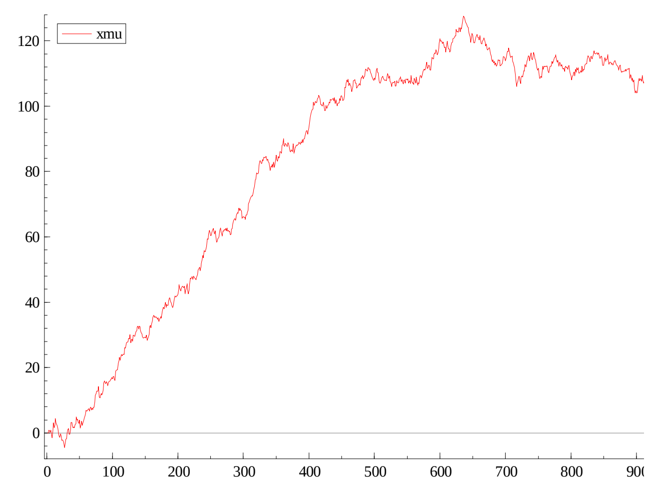

Fig. 3 A Stochastic trend

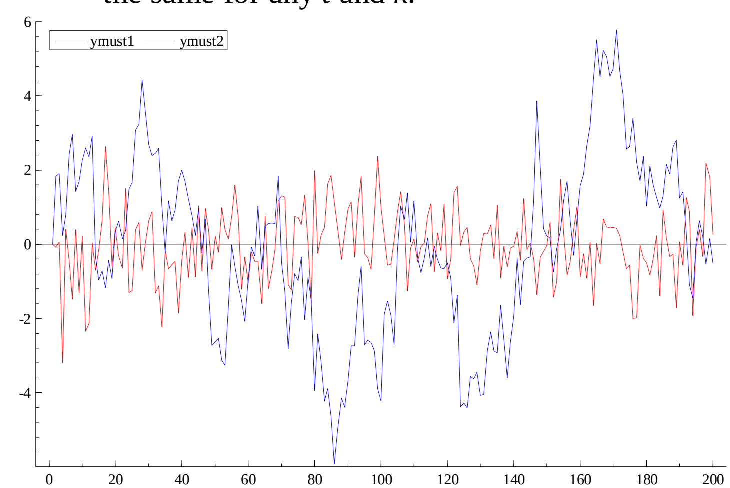

Fig 5 Stationary autoregressive processes

Set up for basic DF test

tests of stationarity are tests of the null hypothesis Ho: $\rho = 1$

The alternative is Ha: $\rho < 1$

Y_{t-1} subtracted from each side

$$\begin{aligned} Y_t - Y_{t-1} &= \rho Y_{t-1} - Y_{t-1} + u_t \\ \Delta Y_t &= (\rho - 1) Y_{t-1} + u_t \\ \Delta Y_t &= \delta Y_{t-1} + u_t \end{aligned} \tag{22}$$

Ho : $\delta = 0$ in $\Delta Y_t = \delta Y_{t-1} + u_t$ implying Y~I(1)

Ha: $\delta < 0$ in $\Delta Y_t = \delta Y_{t-1} + u_t$ implying Y~I(0)

Augmented Dickey-Fuller test

when Y is I(1), the theoretical t-distribution
does not apply to the statistic $\tau = \hat{\delta} \big/ s.e.(\hat{\delta})$.

the correct distribution for $\tau = \hat{\delta} \big/ s.e.(\hat{\delta})$
under Ho : $\delta = 0$ is the Dickey-Fuller distribution.

there is a family of Dickey-Fuller distributions
depending on kind of I(1) processes

$\tau_{nc}, \tau_c, \tau_{ct}, \tau_{ctt}$ are appropriate respectively for
models with no constant,
models with constant,
models with constant and trend,
models with constant plus trend and trend-squared

Dickey-Fuller significance levels

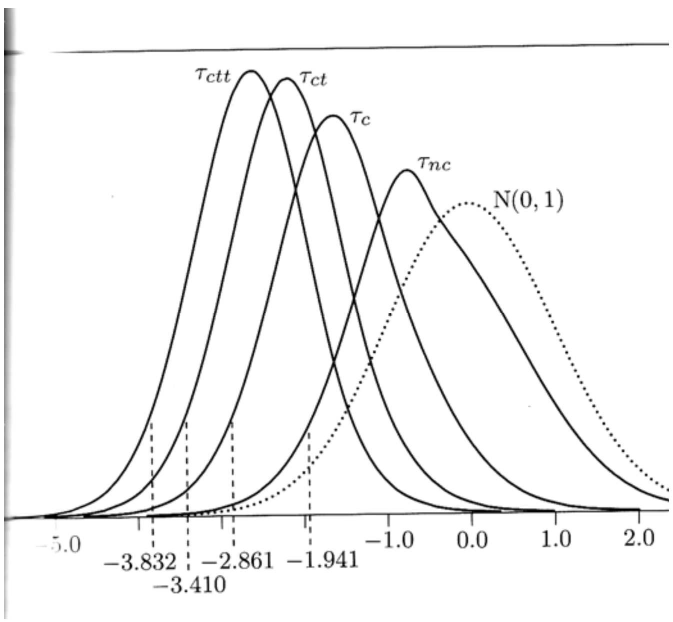

Fig. 8 Asymptotic densities for Dickey-Fuller test statistics

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TABLE 20.4 Critical Values for the Dickey–Fuller DF_τ Test

	Sample Size			
	25	*50*	*100*	∞
F ratio (D–F)[a]	7.24	6.73	6.49	6.25
F ratio (standard)	3.42	3.20	3.10	3.00
AR model[b] (random walk)				
0.01	−2.66	−2.62	−2.60	−2.58
0.025	−2.26	−2.25	−2.24	−2.23
0.05	−1.95	−1.95	−1.95	−1.95
0.10	−1.60	−1.61	−1.61	−1.62
0.975	1.70	1.66	1.64	1.62
AR model with constant (random walk with drift)				
0.01	−3.75	−3.59	−3.50	−3.42
0.025	−3.33	−3.23	−3.17	−3.12
0.05	−2.99	−2.93	−2.90	−2.86
0.10	−2.64	−2.60	−2.58	−2.57
0.975	0.34	0.29	0.26	0.23
AR model with constant and time trend (trend stationary)				
0.01	−4.38	−4.15	−4.04	−3.96
0.025	−3.95	−3.80	−3.69	−3.66
0.05	−3.60	−3.50	−3.45	−3.41
0.10	−3.24	−3.18	−3.15	−3.13
0.975	−0.50	−0.58	−0.62	−0.66

[a] From Dickey and Fuller (1981, p. 1063). Degrees of freedom are 2 and $T - p - 3$.
[b] From Fuller (1976, p. 373 and 1996, Table 10.A.2).

Some alternative DGPs

Pure random walk (y)

$$Y_t = \rho Y_{t-1} + u_t$$

$$\rho = 1$$

$$u \sim N(0, \sigma^2)$$

random walk with drift (ymu)

$$Y_t = Y_{t-1} + \mu + u_t$$

μ is a constant positive (or negative) contribution at each point in time, the Y series drifts consistently upwards (or downwards)

TSP (yt)

$$Y_t = \rho Y_{t-1} + \mu + \gamma_1 t + u_t$$

$$\rho < 1$$

Stationary autoregressive Process (ys)

$$Y_t = \mu + \rho Y_{t-1} + u_t$$

$$\rho < 1$$

Alternative DGPs

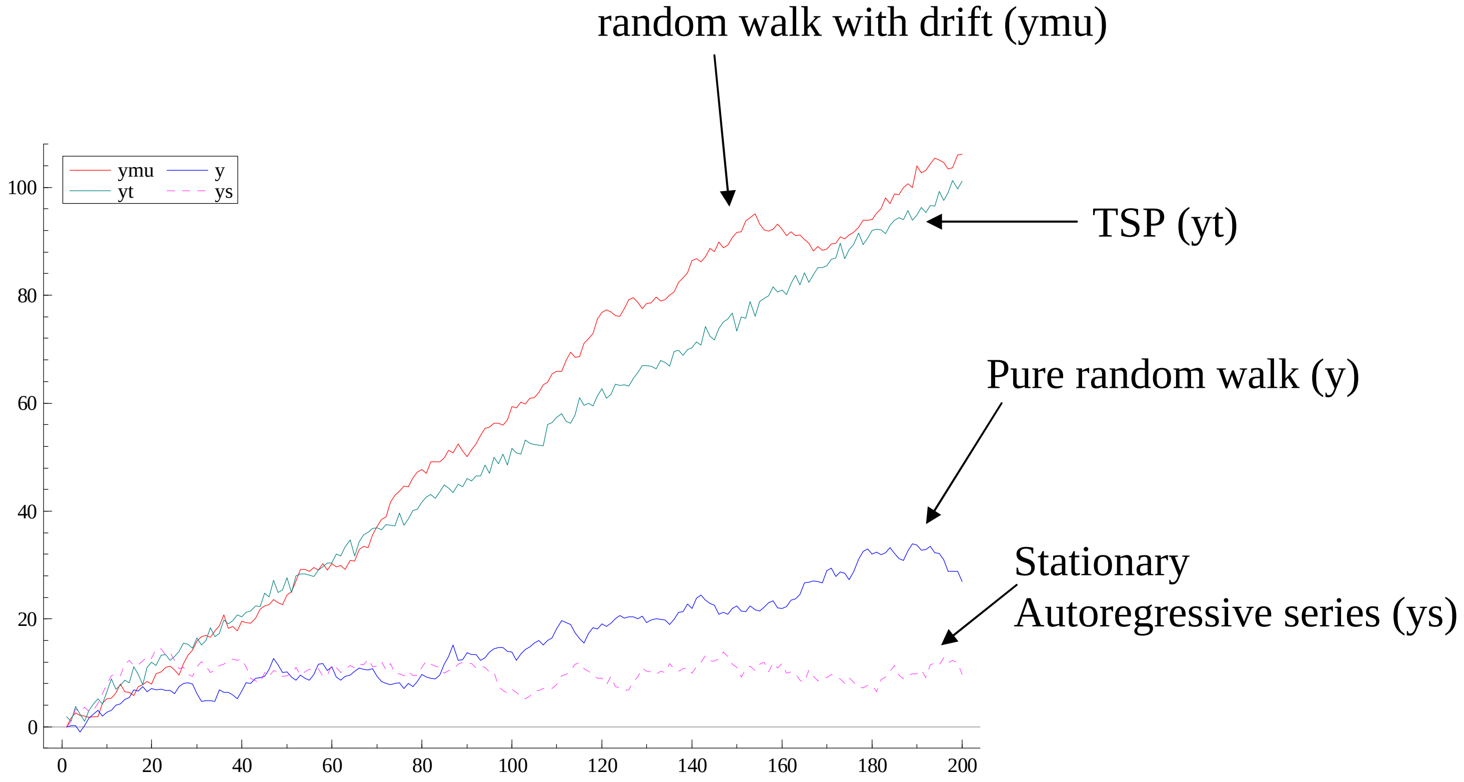

Fig 7 Deterministic and stochastic trends

Augmented Dickey-Fuller test (ADF)

We test each data generating process against an alternative,
To see if the DF test picks out the stationary from the nonstationary ones

the errors in these regressions may not be independent, 'so-called 'white noise', but may in fact be autocorrelated. We therefore can whiten them by including lagged differences in the equation, so as to eliminate serial correlation from the errors.

linear time trend plus constant, augmented by lagged differences

$$\Delta Y_t = \mu + \delta Y_{t-1} + \gamma t + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + ... + u_t \qquad (30)$$

ADF for random walk with drift (ymu)

Results from UNITROOT_tests.fl

```
Augmented Dickey-Fuller test for ymu; regression of Dymu on:

               Coefficient    Std.Error      t-value
ymu_1           -0.0047175    0.0035605      -1.3250
Constant           0.46391     0.066725       6.9526
Trend            0.0024519    0.0017815       1.3763
Dymu_1           0.0063304     0.031837      0.19884
Dymu_2           -0.015645     0.031837     -0.49142

sigma = 0.988925  DW = 1.997  DW-ymu = 5.974e-005  ADF-ymu = -1.325
Critical values used in ADF test: 5%=-3.417, 1%=-3.972
RSS = 970.1479611 for 5 variables and 997 observations
```

ADF

- Previous slide was output from PcGive
- It shows critical values that are same as Dickey-Fuller table (see earlier)
- Next slide is from gretl, it gives the p-value from the same reference distribution
- This can be obtained by running adf_gretl.inp script
- Note that the p-value is NOT from the t distribution, which can be checked in gretl using gretl>tools>p-value finder

ADF for random walk with drift (ymu)

```
Augmented Dickey-Fuller test for ymu
including 2 lags of (1-L)ymu
sample size 997
unit-root null hypothesis: a = 1

   with constant and trend
   model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: -0.000
   lagged differences: F(2, 992) = 0.141 [0.8687]
   estimated value of (a - 1): -0.00471751
   test statistic: tau_ct(1) = -1.32496
   asymptotic p-value 0.8816

Augmented Dickey-Fuller regression
OLS, using observations 4-1000 (T = 997)
Dependent variable: d_ymu

             coefficient   std. error   t-ratio    p-value
  --------------------------------------------------------
  const       0.463909     0.0667246     6.953    6.50e-012 ***
  ymu_1      -0.00471751   0.00356050   -1.325    0.8816
  d_ymu_1     0.00633042   0.0318374     0.1988   0.8424
  d_ymu_2    -0.0156454    0.0318374    -0.4914   0.6232
  time        0.00245187   0.00178151    1.376    0.1690
```

ADF for Pure random walk (y)

```
Augmented Dickey-Fuller test for y
including 2 lags of (1-L)y
sample size 997
unit-root null hypothesis: a = 1

   with constant and trend
   model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: -0.001
   lagged differences: F(2, 992) = 2.476 [0.0846]
   estimated value of (a - 1): -0.0207408
   test statistic: tau_ct(1) = -3.2372
   asymptotic p-value 0.07718

Augmented Dickey-Fuller regression
OLS, using observations 4-1000 (T = 997)
Dependent variable: d_y

             coefficient    std. error    t-ratio   p-value
  ---------------------------------------------------------
  const      -0.278490      0.0859898     -3.239    0.0012  ***
  y_1        -0.0207408     0.00640702    -3.237    0.0772  *
  d_y_1      -0.0357528     0.0317352     -1.127    0.2602
  d_y_2       0.0586645     0.0317069      1.850    0.0646  *
time       -0.000367871   0.000185549   -1.983    0.0477  **
```

ADF for Trend Stationary Process (yt)

```
Augmented Dickey-Fuller test for yt
including 2 lags of (1-L)yt
sample size 997
unit-root null hypothesis: a = 1

   with constant and trend
   model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: 0.001
   lagged differences: F(2, 992) = 1.477 [0.2289]
   estimated value of (a - 1): -0.931959
   test statistic: tau_ct(1) = -17.6711
   asymptotic p-value 5.533e-057

Augmented Dickey-Fuller regression
OLS, using observations 4-1000 (T = 997)
Dependent variable: d_yt

             coefficient   std. error   t-ratio    p-value
  ---------------------------------------------------------
  const       1.01543     0.0675249     15.04     3.40e-046 ***
  yt_1       -0.931959    0.0527391    -17.67     5.53e-057 ***
  d_yt_1     -0.0298141   0.0441098     -0.6759   0.4993
  d_yt_2      0.0211984   0.0317835      0.6670   0.5050
  time        0.465897    0.0263646     17.67     5.75e-061 ***
```

Stationary Autoregressive series (ys)

```
Augmented Dickey-Fuller test for ys
including 2 lags of (1-L)ys
sample size 997
unit-root null hypothesis: a = 1

   with constant and trend
   model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
   1st-order autocorrelation coeff. for e: -0.000
   lagged differences: F(2, 992) = 0.552 [0.5759]
   estimated value of (a - 1): -0.108455
   test statistic: tau_ct(1) = -7.63655
   asymptotic p-value 2.928e-011

Augmented Dickey-Fuller regression
OLS, using observations 4-1000 (T = 997)
Dependent variable: d_ys

             coefficient    std. error    t-ratio    p-value
  ---------------------------------------------------------
  const       1.10259       0.152919       7.210    1.11e-012 ***
  ys_1       -0.108455      0.0142021     -7.637    2.93e-011 ***
  d_ys_1      0.0283473     0.0315447      0.8986   0.3691
  d_ys_2      0.0189067     0.0314805      0.6006   0.5483
time       -4.61587e-05   0.000111105   -0.4155   0.6779
```

Augmented Dickey-Fuller test (ADF)

the ADF test successfully <u>discriminates</u> between

the two <u>stationary series</u>

Trend Stationary Process (yt)
Stationary autoregressive series (ys),

and the two <u>I(1) series</u>

Pure random walk (y)
random walk with drift (ymu).

Augmented Dickey-Fuller test (ADF)

if the null was stationarity rather than nonstationarity- simpler?

contradict the findings of the conventional DF tests,
but also tend to contradict each other!

The majority of the (large number of) unit root tests have specified Ho as a unit root rather than stationarity.

Type I and II error

state of world	decision accept Ho	decision Ho rejected
Ho true	correct Prob=1-α	Type I error Prob=α
H_A true	Type II error Prob=β	correct Prob = 1-β = power

Augmented Dickey-Fuller test (ADF)

problems

size distortion

Type I error rate is not equal to what is tabulated

low power

Type II error rate is high

Even when the null of a unit root is not true, the ADF often fails to reject the null, so we think we have a unit root when we do not.