EC408 Topics in Applied Econometrics

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Rationale for ECMs

Regressing non-stationary variables tends to produce spurious regression

high R-squared, high t values etc even when the X, Y series are totally independent

we can eliminate spurious regression by regressing first differences

, assuming $X \sim \mathrm{I}(1), Y \sim \mathrm{I}(1)$ the regression

$$\begin{aligned} &\Delta Y_t = b_1 \Delta X_t + u_t \\ &u \sim N(0, \sigma^2) \end{aligned} \qquad (31)$$

is a regression of two stationary variables and is therefore devoid of stochastic trend

However equation (31) omits to take account of other effects that may influence ΔY_t and so it may be misspecified

Rationale for ECMs

Assume there is a long-run equilibrium relationship between the levels of *X* and *Y*

- for example consumers' expenditure on average tends to be a fixed proportion of the level of personal disposable income.
- At time t the level of CEX may be less than one would anticipate from the level of PDI
- In the next time period the change in CEX may be greater than expected simply from the change in PDI because there is <u>additional change due to catching up</u> towards the level one would expect.

This leads us to the so-called error correction model (ECM)

Rationale for ECMs

$$\Delta Y_t = b_1 \Delta X_t - b_2 (Y_{t-1} - \hat{Y}_{t-1}) + u_t$$

$$Y_t = \beta_1 X_t + e_t$$

$$\hat{Y}_{t-1} = \hat{\beta}_1 X_{t-1}$$

$$Y_{t-1} - \hat{Y}_{t-1} = \hat{e}_{t-1}$$

The regression $Y_t = \beta_1 X_t + e_t$ is the static levels equation or the long-run equation

The term $Y_{t-1} - \hat{Y}_{t-1} = Y_{t-1} - \hat{\beta}_1 X_{t-1} = \hat{e}_{t-1}$ is the error correction term

a measure of the amount of disequilibrium at time t-1

Rationale for ECMs

If the actual level Y_{t-1} is less than the predicted level $\hat{Y}_{t-1}$ then the error correction term is negative, so we should expect the value of b_2 to be negative so that the overall effect is a positive contribution to ΔY_t. Likewise if Y_{t-1} exceeds $\hat{Y}_{t-1}$, then the error correction term is positive in the next period we should see a negative contribution to ΔY_t.

1. We need to know the specification of the long-run relationship between the levels of *X* and *Y*. Above we simply assumed it was $Y_t = \beta_1 X_t + e_t$. Let us call this the static long run model.
2. We are assuming that the variables in the ECM are stationary, however since *X*~I(1) and *Y* ~I(1), and we are using $Y_{t-1} - \hat{\beta}_1 X_{t-1}$ as a variable in the ECM, this appears to be not the case. Without $Y_{t-1} - \hat{\beta}_1 X_{t-1}$ also being I(0), then equation (32) is not internally consistent.

Cointegration allows this to happen

What cointegration means

X and *Y* are cointegrated if a linear combination of *X* and *Y* is stationary. One linear combination is $e_t = Y_t - \beta_1 X_t$, so if $e \sim I(0)$, then *Y* and *X* are cointegrated.

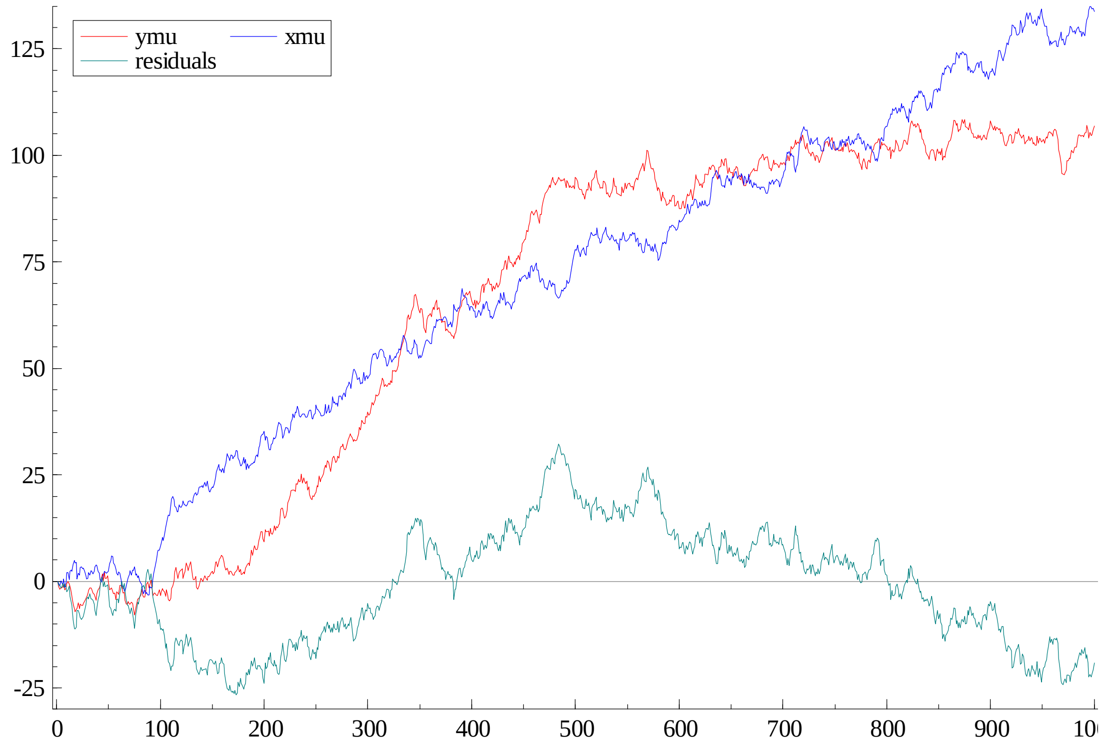

Fig 9 Two I(1) variables and residuals from the regression

What cointegration means

In Fig 9 we see two I(1) variable ymu and xmu. Regressing these we obtain the linear combination $\hat{e}$ = ymu - 0.942441*xmu = residuals, hence $\hat{\beta}_1$ =0.942441 .

the residuals from this static long run model appears to be stationary

while both X and Y are I(1), they appear to be on the same wavelength. For example CEX and PDI may be I(1) variables, but they move in harmony or are synchronized random walks. So their linear combination, the residuals, has an absence of stochastic trend

Using the ADF to test for cointegration

testing for cointegration amounts to testing that the static long run model residuals e ~ I(0), and we can use the DF test, or the ADF test

$$\Delta\hat{e}_t = \mu + \delta\hat{e}_{t-1} + \gamma_1\Delta\hat{e}_{t-1} + \gamma_2\Delta\hat{e}_{t-2} + ... + \gamma_p\Delta\hat{e}_{t-p} + \omega_t \qquad (33)$$

$$\omega_t \sim IID(0, \sigma^2)$$

the standard ADF representation (equation 30) applied to the $\hat{e}_t$

Ho: $\delta = 0$ which states that the residuals have a unit root and therefore we do not have cointegration

$\tau = \hat{\delta} \big/ s.e.(\hat{\delta})$ and refer this to the appropriate ADF distribution

Using the ADF to test for cointegration

a <u>two stage process</u>. the <u>Engle-Granger procedure</u>

- First fit the static long run model, test the residuals *e* for stationarity using ADF.
- Second, If we do have cointegration for the long run regression variables, plug the residuals in as an error correction term in the ECM.

Using the ADF to test for cointegration

Difficulties

1) when T is relatively small, the estimate of the static long run regression coefficient β_1, and hence $\hat{e}$, is biased.

- As $T \to \infty$ on the other hand, $\hat{\beta}_1 \to \beta_1, bias \to 0$. So we have consistent estimates of β_1 but small sample bias
- superconsistency. As $T \to \infty$, the OLS estimator $\hat{\beta}_1 \to \beta_1$ faster than the OLS estimator for I(0) variables

we do not have to worry about which if any of the variables is endogenous. Normally, using OLS with an endogenous left hand side variable results in estimates that are biased and inconsistent. With superconsistency we do not have to worry about this, as $T \to \infty$, the I(1) variables come to dominate the I(0) variable (*e*) and the fact that *e* contains the bias due to endogeneity and autocorrelation due to any omitted dynamic terms (X_{t-1}, Y_{t-1}) matters less also.

- the t statistic does not approximate to the theoretical Student t distribution

Using the ADF to test for cointegration

2) Given $\hat{e}$, the test of Ho: $\delta = 0$ using $\tau = \hat{\delta} / s.e.(\hat{\delta})$ is not straightforward.

- the standard significance values in Table 20.4/Fig 8 that apply to a single variable such as X are not generally applicable to residuals $\hat{e}$.

we have used OLS to obtain $\hat{e}$, and the $\hat{e}$ have been chosen to minimize the residual variance $\hat{\sigma}^2$. So the estimation method is making the $\hat{e}$ as stationary as possible. This means that the standard tables would tend to over reject Ho.

Using the ADF to test for cointegration

The distribution of $\tau = \hat{\delta} / s.e.(\hat{\delta})$ when Ho: $\delta = 0$ is true depends on the number of regressors (up to now we have only had one in our long run static equation), sample size T and whether a constant or trend is included in the ADF equation. Fortunately an equation has been created by Mackinnon which calculates the critical values, and this is embodied within the standard software.

This is

$$\tau_{crit,0.05} = \phi_{\infty} + \frac{\phi_1}{T} + \frac{\phi_2}{T^2} \tag{34}$$

And for just one regressor the relevant values for 5% significance are

$\phi_{\infty} = -2.8621, \phi_1 = -2.738, \phi_2 = -8.36$

Using the ADF to test for cointegration

Different values apply if we want 1% or 10% significance.

Also if we also include a constant in our levels model then

$\phi_{\infty} = -1.9393, \phi_1 = -0.398, \phi_2 = 0$

The values are also different if we have more than one regressor in the levels model (we have assumed only 1 thus far).

Notice that with $T \to \infty$, $\tau_{crit,0.05}$ is as given in Figure 8, but this is only the case when we are using just two I(1) variables to generate the residuals $\hat{e}$.

Clearly, in general we should use a computer to work out the appropriate critical values to test whether the $\hat{e}$ are stationary and therefore whether we have cointegration. Fortunately standard software gives the critical values automatically.

Using the ADF to test for cointegration

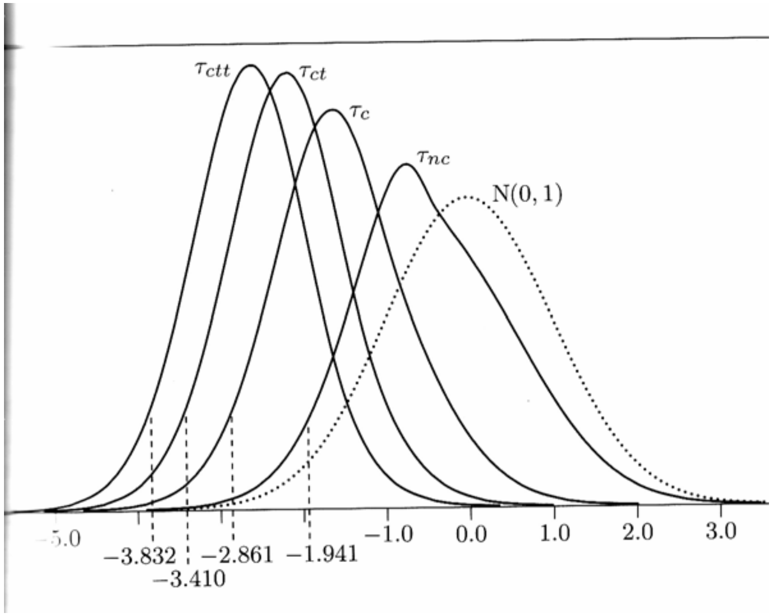

Fig. 8 Asymptotic densities for Dickey-Fuller test statistics