EC408 Topics in Applied Econometrics

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Applied Econometrics

- What is spurious regression?
- How do we check for stochastic trends?
- Cointegration and Error Correction Models
- Autoregressive distributed lag (ADL) models
- VAR models

Applied Econometrics

- What is spurious regression?
  - Stationarity
  - Deterministic trends (TSPs)
  - Stochastic trends
  - Implications of nonstationarity for regression
  - Differencing to induce stationarity

What is spurious regression?

False association between variables due to a common trend

R-squared= 0.97

t-ratio = 41.6

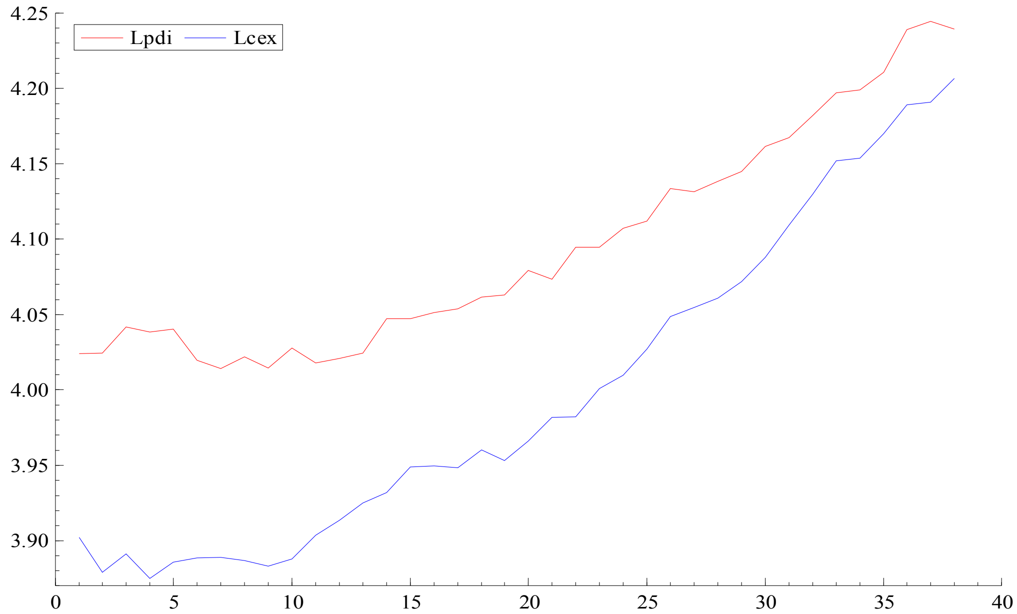

Fig. 1. Ln PDI versus Ln consumers' expenditure over 40 years

The regression picks up the underlying common trend, to give a false Impression of the true relationship. If trend eliminated, this would reveal the true association between log CEX and log PDI.

What is spurious regression?

- problem was acknowledged but the method of dealing with it inadequate
- field implicitly assumed that economic data were trend stationary processes (TSPs)
- if a series was trending upwards through time, it could easily be detrended by removing the deterministic trend
- It was imagined that one would be left with a stationary variable, with a constant mean and variance – not true!
- Stationarity is the 'sine qua non' of regression analysis, but most economic variables are not TSPs

Stationarity

for Y_t, $t = 1,\ldots,T$,

$$E(Y_t) = \mu$$
$$\mathrm{var}(Y_t) = \sigma^2 \qquad (1)$$
$$\mathrm{cov}(Y_t, Y_{t+j}) = \rho_j$$

mean and variance are constant over time

covariance of two observations *j* units of time apart is the same for any value of *t*

Deterministic trends (TSPs)

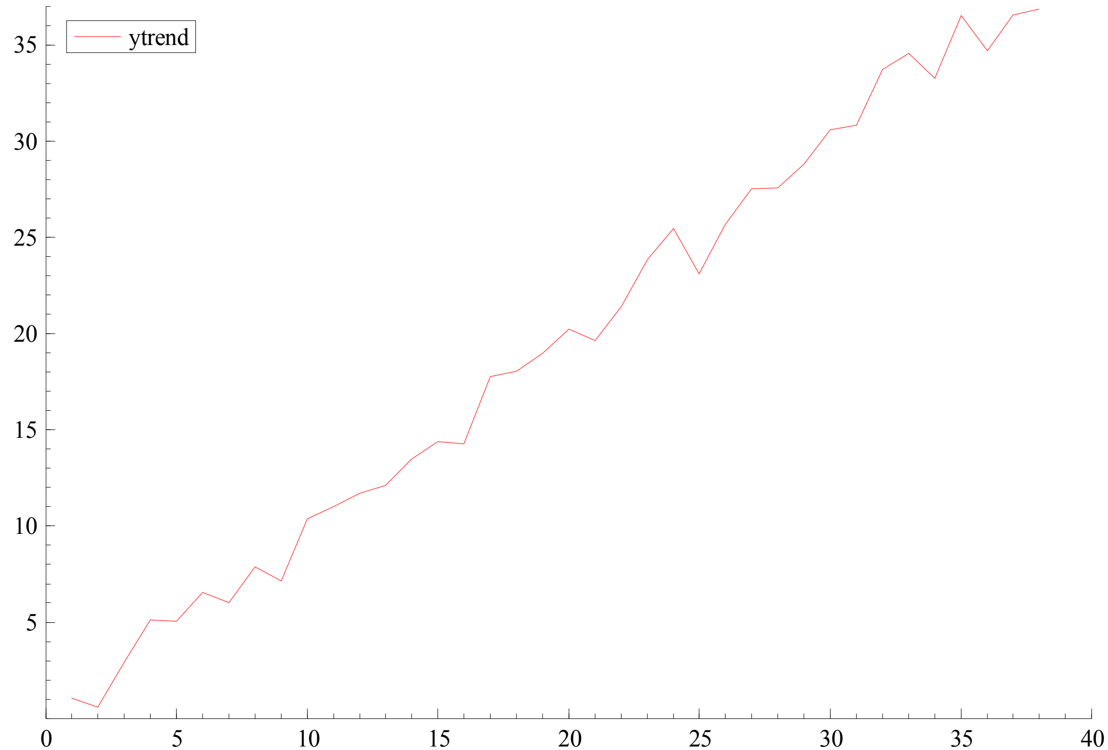

Fig. 2 Deterministic Trend

the trend is perfectly predictable

Y fluctuates randomly about the trend line. If it were not for the trend, *Y* would be stationary

Deterministic trends (TSPs)

eliminate a deterministic trend

Introduce time as an explicit regression variable

$$Y = b_0 + b_1 X + b_t t + u \qquad (2)$$

$$u \sim IID(0, \sigma^2)$$

The ~ here means 'is distributed as'

IID stands for Independent and Identically Distributed

at each point in time t the error is drawn from the same probability distribution, with 0 mean, and the same variance

Deterministic trends (TSPs)

```
EQ( 2) Modelling Lcex by OLS (using CEXPDI.xls)
       The estimation sample is: 1 to 38

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              -1.83200     0.1403    -13.1   0.000   0.8258
Lpdi                   1.42357    0.03425     41.6   0.000   0.9796

sigma               0.0152556  RSS              0.0083784056
R^2                  0.979592  F(1,36) =      1728 [0.000]**
log-likelihood        106.054  DW                      0.959
no. of observations        38  no. of parameters           2
mean(Lcex)            3.99731  var(Lcex)           0.0108038

Lcex =  - 1.832 + 1.424*Lpdi
(SE)     (0.14)  (0.0342)
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Deterministic trends (TSPs)

$$Lcex = b_0 + b_1 Lpdi + b_2 t + u \qquad (3)$$

$$u \sim N(0, \sigma^2)$$

```
EQ( 3) Modelling Lcex by OLS (using CEXPDI.xls)
       The estimation sample is: 1 to 38

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -0.293163     0.3015   -0.972   0.338   0.0263
Lpdi                  1.03482    0.07586     13.6   0.000   0.8417
Trend              0.00272102  0.0004999     5.44   0.000   0.4584

sigma               0.0113864  RSS             0.00453773252
R^2                  0.988947  F(2,35) =      1566 [0.000]**
log-likelihood        117.706  DW                       1.19
no. of observations        38  no. of parameters           3
mean(Lcex)            3.99731  var(Lcex)           0.0108038

Lcex =  - 0.2932 + 1.035*Lpdi + 0.002721*Trend
(SE)     (0.301)  (0.0759)     (0.0005)
```

The estimated regression coefficient with the trend term in the regression is much lower, with 1% change in PDI causing 1.035% change in CEX. However, the DW statistic is still somewhat low, and is still below the critical value of 1.53 needed to retain the null of no residual autocorrelation.

Stochastic trends

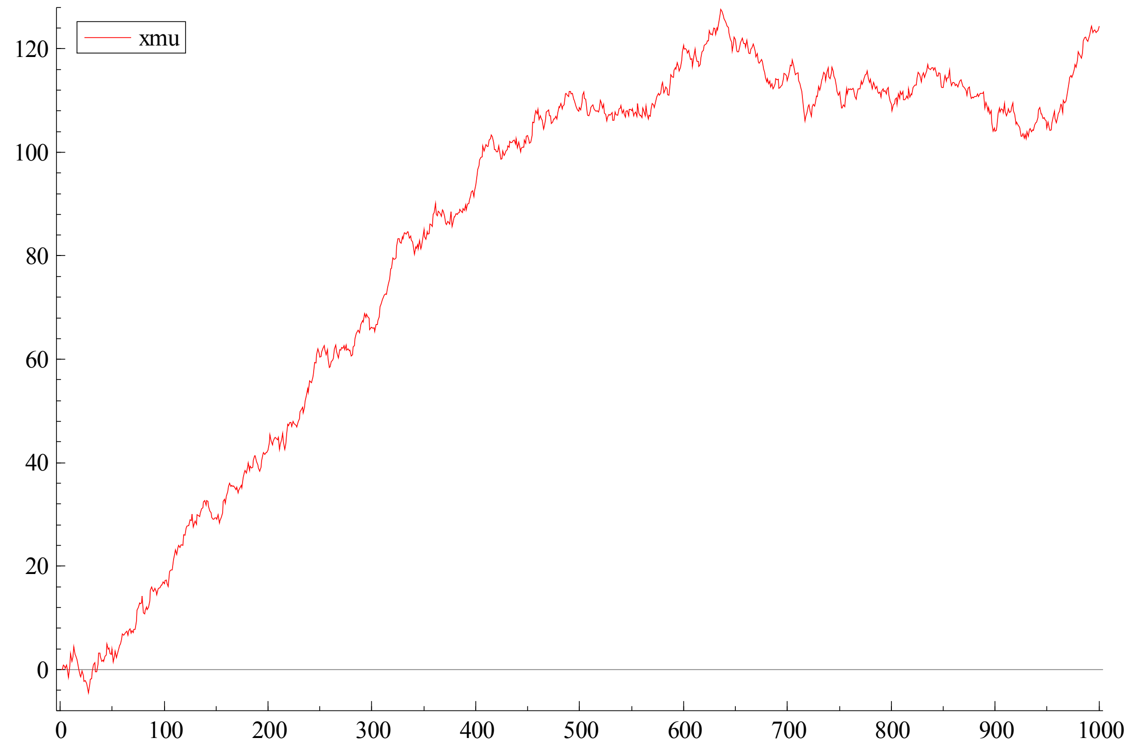

Fig. 3 A Stochastic trend

A stochastic trend is not perfectly predictable

Stochastic trends

- forecast from a stochastic trend are much less reliable
- if we think our trend is deterministic when in fact it is stochastic, our long term forecasts are likely to be very wrong.

Stochastic trends

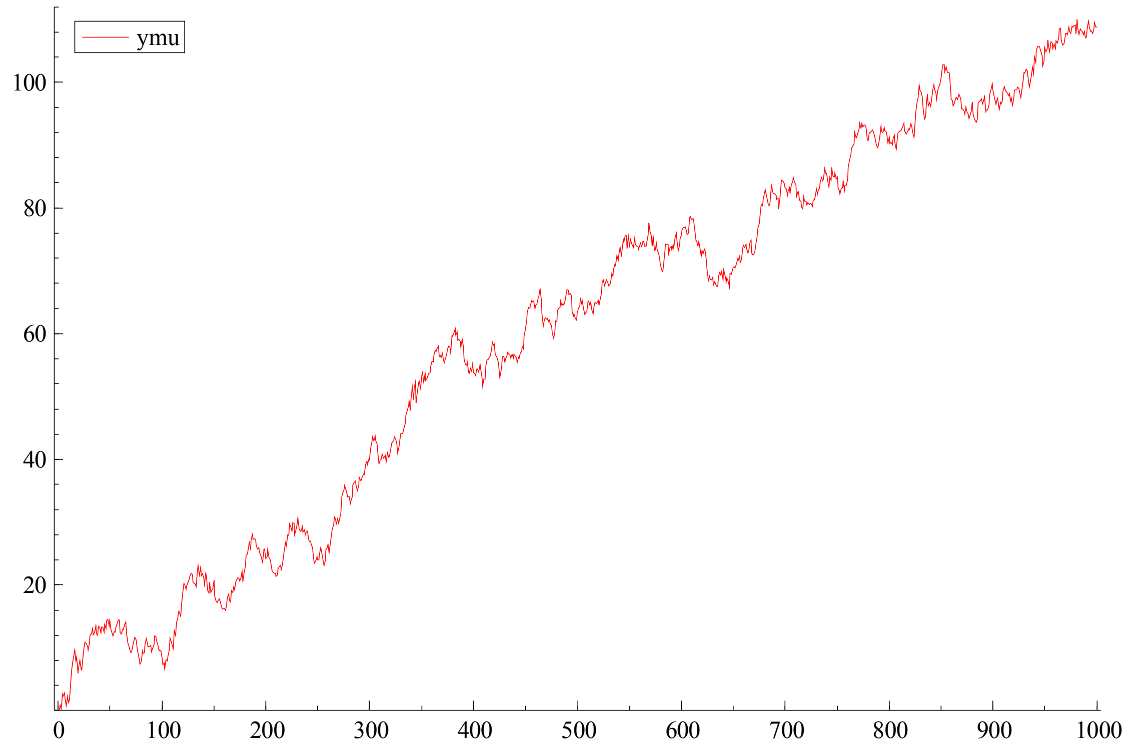

Fig 4 An apparent TSP which is actually a Stochastic trend

Stochastic trends

both Figs 3 and 4 were generated in exactly the same way, as random walks

A random walk occurs when the value of Y_t is equal to Y_{t-1} plus a random shock u_t

$$Y_t = Y_{t-1} + u_{1t} \qquad (5)$$
$$u_{1t} \sim N(0, \sigma_1^2)$$

The random walk process ensures that the variance of Y Is not constant, but increases

Stochastic trends

$$
\begin{aligned}
Y_0 &= 0 \\
Y_1 &= Y_0 + u_{11} \\
Y_2 &= Y_1 + u_{12} = Y_0 + u_{11} + u_{12} \\
Y_3 &= Y_2 + u_{13} = Y_0 + u_{11} + u_{12} + u_{13} \\
&\bullet \\
&\bullet \\
&\bullet \\
Y_T &= Y_{T-1} + u_{1T} = Y_0 + u_{11} + u_{12} + u_{13} + \ldots\ldots + u_{1T}
\end{aligned}
\qquad (6)
$$

$$
or \qquad Y_T = Y_0 + \sum_{i=1}^{T} u_{1i}
$$

$$
E(Y_T) = Y_0 \qquad \mathrm{var}(Y_T) = T\sigma_1^2
$$

at point T in time, the variance of the process is T times the variance of the shocks

Stochastic trends

Stochastic trend: I(1) process

$$Y_t = \rho Y_{t-1} + u_{1t} \qquad (7)$$
$$\rho = 1$$
$$u_{1t} \sim N(0, \sigma_1^2)$$

the impact of a shock
does not dies out,
it is permanent

Stationary process

$$Y_t = \rho Y_{t-1} + u_{1t} \qquad (8)$$
$$\rho < 1$$
$$u_{1t} \sim N(0, \sigma_1^2)$$

$$Y_T = \rho^T Y_0 + \sum_{i=0}^{T-1} \rho^i u_{1i} \qquad (9)$$

influence of a shock transitory,
dying out as time passes

Stochastic trends

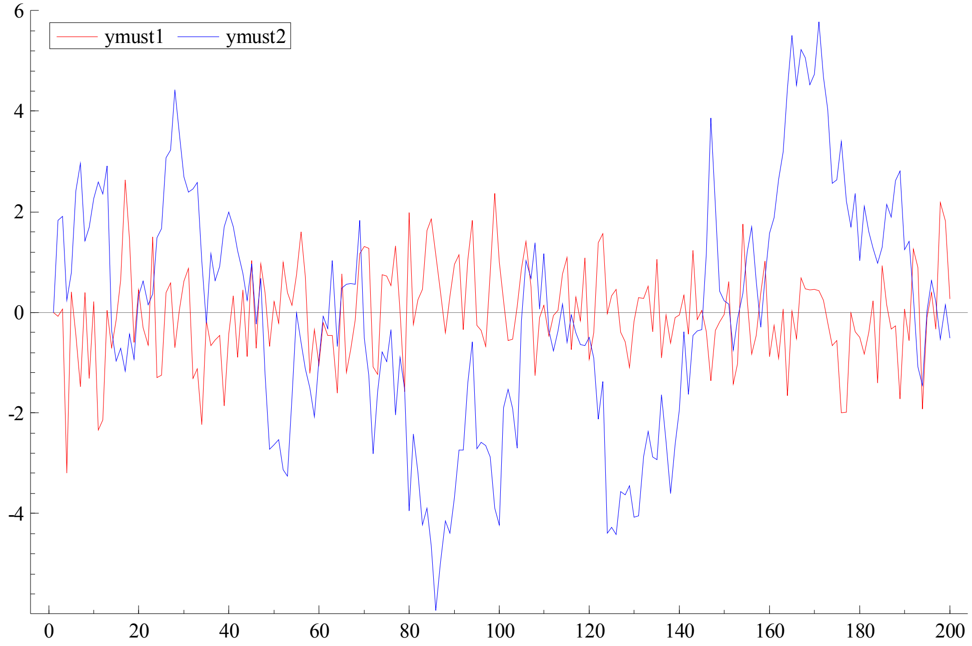

Fig 5 Stationary autoregressive processes

two autoregressive series with $\rho = 0.2$ and 0.9

the value at time t remembers to some extent the value at t-1,

but both series are stationary.

Implications of nonstationarity for regression

$$Y_t = Y_{t-1} + u_{1t}$$
$$u_{1t} \sim N(0, \sigma_1^2) \qquad (10)$$

$$X_t = X_{t-1} + u_{2t}$$
$$u_{2t} \sim N(0, \sigma_2^2) \qquad (11)$$

$$Y_t = b_0 + b_1 X_t + e_t \qquad (12)$$

the two series are independent,
anticipate that $\hat{b}_1$ will not usually differ significantly from Ho: $b_1 = 0$
R-squared will be around 0

Implications of nonstationarity for regression

But

regression of independent and nonstationary variables

characterized by

- a (very) high R-squared
- (very) high individual t-statistics
- a low Durbin Watson statistic

1)distributions of t-stats, F-stats, and R-squared are non-standard.

2)With larger samples the null of no relationship is likely to be rejected more frequently – rejection rates increase with sample size.

<u>When regressions involve non-stationary variables, the estimation results should not be taken too seriously. (Granger and Newbold, 1974).</u>

Implications of nonstationarity for regression

Many regression quantities depend on

$$S_{XY} = \sum XY - \sum X \sum Y / T \quad = \sum_{t=1}^{T} (X_t - \bar{X})(Y_t - \bar{Y})$$

$$S_{XX} = \sum X^2 - (\sum X)^2 / T \quad = \sum_{t=1}^{T} (X_t - \bar{X})^2 \qquad (14)$$

$$S_{YY} = \sum Y^2 - (\sum Y)^2 / T \quad = \sum_{t=1}^{T} (Y_t - \bar{Y})^2$$

If X,Y are stationary I(0) variables as T gets larger S_{XX}, S_{YY} and S_{XY} increase at a rate such that S_{XX}/T, S_{YY}/T and S_{XY}/T tend to finite stable quantities.

When X,Y are I(1), the rate of increase of S_{XX}, S_{YY} and S_{XY} as T increases is faster so that there is no convergence to finite stable quantities

Implications of nonstationarity for regression

For example

$$t = \frac{\hat{b}_1}{\hat{\sigma} / \sqrt{S_{XX}}}$$

$$\hat{b}_1 = S_{XY} / S_{XX}$$

$$\hat{\sigma}^2 = (S_{YY} - \hat{b}_1 S_{XY}) / (T - 2) \qquad (13)$$

Differencing to induce stationarity

$$Y_t = Y_{t-1} + u_{1t}$$
$$u_{1t} \sim N(0, \sigma_1^2) \longleftarrow \text{stationary} \tag{10}$$

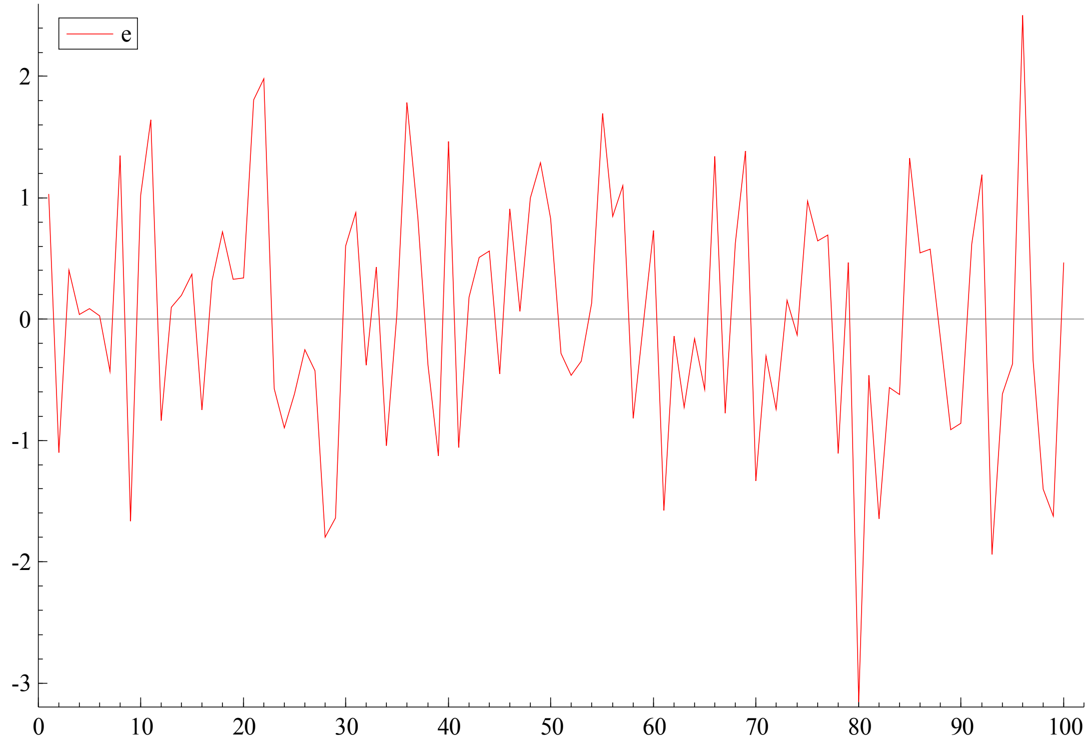

Fig 6 A stationary series (drawn from a Normal distribution)

Differencing to induce stationarity

$$Y_t \sim \mathrm{I}(1) \therefore \Delta Y_t = Y_t - Y_{t-1} \sim \mathrm{I}(0)$$
$$Y_t \sim \mathrm{I}(2) \therefore \Delta Y_t = Y_t - Y_{t-1} \sim \mathrm{I}(1) \qquad (18)$$
$$(\Delta Y_t) - (\Delta Y_{t-1}) \sim \mathrm{I}(0)$$

$$Y_t \sim \mathrm{I}(1), X_t \sim \mathrm{I}(1)$$
$$u_{1t} = Y_t - Y_{t-1} = \Delta Y_t \sim \mathrm{I}(0) \qquad (17)$$
$$u_{2t} = X_t - X_{t-1} = \Delta X_t \sim \mathrm{I}(0)$$

$$\Delta Y_t = b_0 + b_1 \Delta X_t + u_t \qquad (19)$$
$$u \sim N(0, \sigma^2)$$

Using differenced variables in the regression means that we may have eliminated stochastic trend. Note also that the presence of the constant in the difference equation means that we are assuming a time trend in the equivalent levels equation.

Differencing to induce stationarity

```
EQ( 7) Modelling DLcex by OLS (using CEXPDI.xls)
       The estimation sample is: 2 to 38

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant           0.00629329   0.001897     3.32   0.002   0.2392
DLpdi                0.331504     0.1623     2.04   0.049   0.1065

sigma               0.0100083  RSS               0.00350580537
R^2                  0.106453  F(1,35) =       4.17 [0.049]*
log-likelihood        118.888  DW                         1.87
no. of observations        37  no. of parameters             2
mean(DLcex)        0.00822234  var(DLcex)           0.00010604

DLcex =  + 0.006293 + 0.3315*DLpdi
(SE)       (0.0019)   (0.162)
```

A 1% increase in PDI produces only 0.33% increase in CEX

This is somewhat different from what we found by treating CEX and PDI as TSPs rather that as having stochastic trends. The reaction of CEX is much less to changes in PDI, and the significance of the relationship is much lower

the constant is significantly different from zero, indicating significant autonomous growth in CEX, which is growing regardless of PDI growth

DW = 1.87 is well above the upper bound given by the D-W tables of 1.53

Differencing to induce stationarity

the model does not tell the whole story regarding the relationship between PDI and CEX. The growth relationship may in the long run predict excessively high (low) levels of CEX, which are known to be very different from what one would expect given the level of PDI.

We have seen that stochastic trends lead to spurious regression, and this leads us to use differenced variables. But a regression of differenced variables ignores the long-run relationship between the levels of the variables, and so the results obtained may still be questionable

ultimately we take account of the short-run relationship between $\Delta X, \Delta Y$, but also correct for deviations this produces from a long-run relationship between the levels X, Y