EC408 Topics in Applied Econometrics

B Fingleton, Dept of Economics, Strathclyde University

Model reduction, F and LR tests to choose lags

The F test is a very useful test statistic for assessing the importance of a group of variables.

- Given two models, one with and one without the group of variables to be tested
  - it calculates the change in the level of explanation per parameter (or degree of freedom)
  - and compares it with the amount of random variation per degree of freedom in the model.
  - For the group of variables to have had a significant effect, we should expect the change in explanation per additional degree of freedom to exceed the amount of random variation per degree of freedom in the model.

$$\frac{D_1 - D_2}{k'}$$

$$\frac{D_2}{T - k}$$

$$F = \frac{\dfrac{D_1 - D_2}{k'}}{\dfrac{D_2}{T - k}}$$

Model reduction, F and LR tests to choose lags

- D_1 is the unexplained variation (the residual sum of squares) for model 1
- D_2 is the residual sum of squares for model 2
- model 2 is the same as model 1 plus the additional k' explanatory variables (the lags to be tested)
- then $D_1 > D_2$ and the additional explanation is the reduction in RSS, and per degree of freedom lost this is

$$\frac{D_1 - D_2}{k'} \qquad (61)$$

- This quantity is compared with the random variation per model degree of freedom ($T - k$), given by

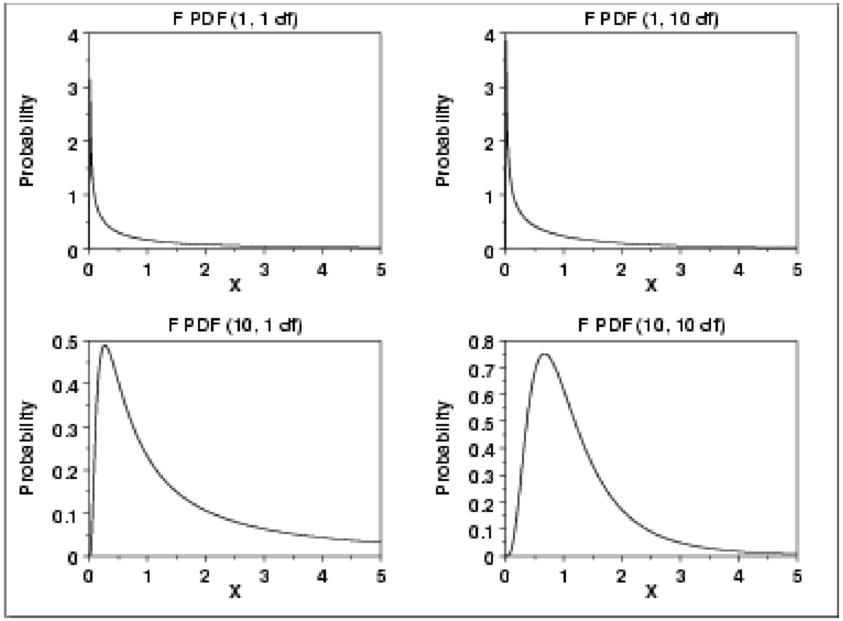
$$\frac{D_2}{T - k} \qquad (62)$$

Model reduction, F and LR tests to choose lags

- Assuming that D_2 is calculated from white noise residuals.
- Assuming that the additional k' explanatory variables (lags) are in fact null (have no effect), then the ratio

$$F = \frac{\frac{D_1 - D_2}{k'}}{\frac{D_2}{T - k}} \sim F_{k',T-k} \qquad (63)$$

F-distribution



- **The F-distribution is simply the ratio of two chi-squared**
- **variables divided by their respective degrees of freedom i.e.**

$$\frac{C_1/d_1}{C_2/d_2} = F(d_1, d_2) \qquad 0 \le F \le \infty$$

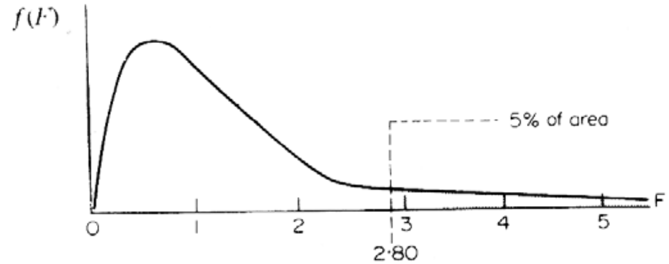


Table 4A. Values of $F_{0\cdot 05,\,\nu_1,\,\nu_2}$

Example
For $\nu_1 = 9$, $\nu_2 = 12$ degrees of freedom
$P(F > 2\cdot 80) = 0\cdot 05$

ν_1 = degrees of freedom for numerator

ν_2 = degrees of freedom for denominator

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18·5	19·0	19·2	19·2	19·3	19·3	19·4	19·4	19·4	19·4	19·4	19·4	19·4	19·5	19·5	19·5	19·5	19·5	19·5
3	10·1	9·55	9·28	9·12	9·01	8·94	8·89	8·85	8·81	8·79	8·74	8·70	8·66	8·64	8·62	8·59	8·57	8·55	8·53
4	7·71	6·94	6·59	6·39	6·26	6·16	6·09	6·04	6·00	5·96	5·91	5·86	5·80	5·77	5·75	5·72	5·69	5·66	5·63
5	6·61	5·79	5·41	5·19	5·05	4·95	4·88	4·82	4·77	4·74	4·68	4·62	4·56	4·53	4·50	4·46	4·43	4·40	4·37
6	5·99	5·14	4·76	4·53	4·39	4·28	4·21	4·15	4·10	4·06	4·00	3·94	3·87	3·84	3·81	3·77	3·74	3·70	3·67
7	5·59	4·74	4·35	4·12	3·97	3·87	3·79	3·73	3·68	3·64	3·57	3·51	3·44	3·41	3·38	3·34	3·30	3·27	3·23
8	5·32	4·46	4·07	3·84	3·69	3·58	3·50	3·44	3·39	3·35	3·28	3·22	3·15	3·12	3·08	3·04	3·01	2·97	2·93
9	5·12	4·26	3·86	3·63	3·48	3·37	3·29	3·23	3·18	3·14	3·07	3·01	2·94	2·90	2·86	2·83	2·79	2·75	2·71
10	4·96	4·10	3·71	3·48	3·33	3·22	3·14	3·07	3·02	2·98	2·91	2·85	2·77	2·74	2·70	2·66	2·62	2·58	2·54
11	4·84	3·98	3·59	3·36	3·20	3·09	3·01	2·95	2·90	2·85	2·79	2·72	2·65	2·61	2·57	2·53	2·49	2·45	2·40
12	4·75	3·89	3·49	3·26	3·11	3·00	2·91	2·85	2·80	2·75	2·69	2·62	2·54	2·51	2·47	2·43	2·38	2·34	2·30
13	4·67	3·81	3·41	3·18	3·03	2·92	2·83	2·77	2·71	2·67	2·60	2·53	2·46	2·42	2·38	2·34	2·30	2·25	2·21
14	4·60	3·74	3·34	3·11	2·96	2·85	2·76	2·70	2·65	2·60	2·53	2·46	2·39	2·35	2·31	2·27	2·22	2·18	2·13
15	4·54	3·68	3·29	3·06	2·90	2·79	2·71	2·64	2·59	2·54	2·48	2·40	2·33	2·29	2·25	2·20	2·16	2·11	2·07
16	4·49	3·63	3·24	3·01	2·85	2·74	2·66	2·59	2·54	2·49	2·42	2·35	2·28	2·24	2·19	2·15	2·11	2·06	2·01
17	4·45	3·59	3·20	2·96	2·81	2·70	2·61	2·55	2·49	2·45	2·38	2·31	2·23	2·19	2·15	2·10	2·06	2·01	1·96
18	4·41	3·55	3·16	2·93	2·77	2·66	2·58	2·51	2·46	2·41	2·34	2·27	2·19	2·15	2·11	2·06	2·02	1·97	1·92
19	4·38	3·52	3·13	2·90	2·74	2·63	2·54	2·48	2·42	2·38	2·31	2·23	2·16	2·11	2·07	2·03	1·98	1·93	1·88
20	4·35	3·49	3·10	2·87	2·71	2·60	2·51	2·45	2·39	2·35	2·28	2·20	2·12	2·08	2·04	1·99	1·95	1·90	1·84
21	4·32	3·47	3·07	2·84	2·68	2·57	2·49	2·42	2·37	2·32	2·25	2·18	2·10	2·05	2·01	1·96	1·92	1·87	1·81
22	4·30	3·44	3·05	2·82	2·66	2·55	2·46	2·40	2·34	2·30	2·23	2·15	2·07	2·03	1·98	1·94	1·89	1·84	1·78
23	4·28	3·42	3·03	2·80	2·64	2·53	2·44	2·37	2·32	2·27	2·20	2·13	2·05	2·01	1·96	1·91	1·86	1·81	1·76
24	4·26	3·40	3·01	2·78	2·62	2·51	2·42	2·36	2·30	2·25	2·18	2·11	2·03	1·98	1·94	1·89	1·84	1·79	1·73
25	4·24	3·39	2·99	2·76	2·60	2·49	2·40	2·34	2·28	2·24	2·16	2·09	2·01	1·96	1·92	1·87	1·82	1·77	1·71
30	4·17	3·32	2·92	2·69	2·53	2·42	2·33	2·27	2·21	2·16	2·09	2·01	1·93	1·89	1·84	1·79	1·74	1·68	1·62
40	4·08	3·23	2·84	2·61	2·45	2·34	2·25	2·18	2·12	2·08	2·00	1·92	1·84	1·79	1·74	1·69	1·64	1·58	1·51
60	4·00	3·15	2·76	2·53	2·37	2·25	2·17	2·10	2·04	1·99	1·92	1·84	1·75	1·70	1·65	1·59	1·53	1·47	1·39
120	3·92	3·07	2·68	2·45	2·29	2·18	2·09	2·02	1·96	1·91	1·83	1·75	1·66	1·61	1·55	1·50	1·43	1·35	1·25
∞	3·84	3·00	2·60	2·37	2·21	2·10	2·01	1·94	1·88	1·83	1·75	1·67	1·57	1·52	1·46	1·39	1·32	1·22	1·00

Abridged from M. Merrington and C. M. Thompson, 'Tables of percentage points of the inverted beta (F) distribution', *Biometrika*, vol. 33, 1943, p. 73. By permission of the *Biometrika* trustees.

Model reduction, F and LR tests to choose lags

We should start with a complex model with several lags, which will tend to produce white noise residuals which are then appropriate for hypothesis testing (remember if we have autocorrelation or heteroscedasticity in the errors then standard errors and inference will be biased). This is in line with the top-down philosophy of Hendry.

The first test <u>tests lag m conditional on the presence of all other lags</u>

It asks the question, does lag m carry any explanatory power given the existence of the other lags in the model?

<u>The second test is of the significance of omitted lags up to lag m</u>

Hence we test for 1…m, then 2…m, then 3…m, etc

Model reduction, F and LR tests to choose lags

The first test tests lag m conditional on the presence of all other lags

. Given the model

$$\begin{aligned} Y_t &= \alpha_1 Y_{t-1} + b_{10} X_{1t} + b_{11} X_{1t-1} + b_{12} X_{1t-2} + b_{13} X_{1t-3} + b_{14} X_{1t-4} \\ &+ b_{20} X_{2t} + b_{21} X_{2t-1} + b_{22} X_{2t-2} + b_{23} X_{2t-3} + b_{24} X_{1t-4} + u_t \end{aligned} \qquad (64)$$

There are $k' = 3$ variables for m = 1,

Y_{t-1} or CONS_1, X_{1t-1} or INC_1, and X_{2t-1} or INFLAT_1,

The denominator of the F ratio is $D_2/(T\text{-}k)$ = 177.365821/(155-11) for the 4 lag unrestricted model (4 lags to create white noise residuals) with degrees of freedom $T - k$ = 155-11 = 144, since there are 155 observations (omitting 4 to allow 4 lags) and 11 parameters, as shown by counting the variables in the model.

Evidently lag 1 has significant explanatory power, since F = 172.54 has a p-value close to zero in the theoretical $F_{3,144}$ distribution. The test statistic is very atypical of the reference distribution assumed under the null.

```
EQ(15) Modelling CONS by OLS (using data.in7)
       The estimation sample is: 1954 (1) to 1992 (3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
CONS_1               0.811974    0.03601     22.6   0.000   0.7793
INC                  0.478975    0.02917     16.4   0.000   0.6518
INC_1               -0.291088    0.04081    -7.13   0.000   0.2611
INC_2              -0.0362995    0.03775   -0.961   0.338   0.0064
INC_3               0.0400228    0.03782     1.06   0.292   0.0077
INC_4             -0.00484922    0.03227   -0.150   0.881   0.0002
INFLAT              -0.910971     0.2631    -3.46   0.001   0.0768
INFLAT_1            0.0640396     0.4672    0.137   0.891   0.0001
INFLAT_2           -0.0435876     0.4845  -0.0900   0.928   0.0001
INFLAT_3            -0.316838     0.4554   -0.696   0.488   0.0034
INFLAT_4             0.112654     0.2722    0.414   0.680   0.0012

sigma                 1.10982  RSS                  177.365821
log-likelihood       -230.382  DW                         1.91
no. of observations       155  no. of parameters            11
mean(CONS)            875.653  var(CONS)               183.482


Tests on the significance of each lag
Lag 1         F(3,144) =   172.54 [0.0000]**
Lag 2         F(2,144) =  0.46687 [0.6279]
Lag 3         F(2,144) =  0.78841 [0.4565]
Lag 4         F(2,144) = 0.086535 [0.9172]
```

Clearly the additional explanatory power of lag 2 ($k' = 2$ additional variables), is weak, given that we have the other lags 1,3, and 4 present in the model. The contribution of lag 2 does not differ much from random variation per degree of freedom (the denominator of the F ratio). Likewise, lags 3 and 4 carry no additional explanatory power.

Model reduction, F and LR tests to choose lags

<u>The second test is of the significance of omitted lags up to lag m</u>

$$\begin{aligned} Y_t &= \alpha_1 Y_{t-1} + b_{10} X_{1t} + b_{11} X_{1t-1} + b_{12} X_{1t-2} + b_{13} X_{1t-3} + b_{14} X_{1t-4} \\ &+ b_{20} X_{2t} + b_{21} X_{2t-1} + b_{22} X_{2t-2} + b_{23} X_{2t-3} + b_{24} X_{1t-3} + u_t \end{aligned} \tag{64}$$

We commence with the set of lags 1… 4 inclusive.

- Looking at the four lag model that this involves 9 variables, so the appropriate degrees of freedom for the F test are $k' = 9$ and $T - k = 144$.
- The test statistic 173.84 has a very low p-value in the reference distribution under the null, which is the $F_{k',T-k}$. At least one lag in lags 1…4 is significant.

Model reduction, F and LR tests to choose lags

```
Lag 1 - 4    F(9,144) =   173.84 [0.0000]**
Lag 2 - 4    F(6,144) =  0.46643 [0.8323]
Lag 3 - 4    F(4,144) =  0.44169 [0.7783]
Lag 4 - 4    F(2,144) = 0.086535 [0.9172]
```

Next we test lags 2…4, which involves $k' = 6$ variables and $T - k = 144$. This has a high p-value so the test statistic 0.46643 is typical of the reference distribution $F_{k',T-k}$ indicating that we should not reject the null that the effects of lags 2…4 are no more than random variation. Similarly, testing lags 3…4 and lag 4 retains the null. It appears from this analysis that the first lag alone is significant.

Model reduction, F and LR tests to choose lags

an entirely equivalent procedure can be accomplished by looking at the log likelihoods of nested models

where we have the log likelihoods of two models we wish to compare, a restricted model giving log L_R and an unrestricted model giving log L_U

Under the null that the k restrictions are true, then a good approximation is

$$-2\log\left(\frac{L_R}{L_U}\right) = 2\log\left(\frac{L_U}{L_R}\right) = 2\left\{\log L_U - \log L_R\right\} \sim \chi_k^2 \qquad (65)$$

Model reduction, F and LR tests to choose lags

We check the whether the likelihood ratio test also does not reject the null that lags 2…4 are zero.

If we compare the 4 lag model (log L_U) with the 1 lag model (log L_R) ,

log L_U = -230.382 (it is unrestricted because no restriction have been placed on the parameters, so all 4 lags are present),

log L_R = -231.873 for the 1 lag model (entailing 6 parameters restricted to zero).

Refer 2{ -230.382 +231.873 } = 2.982 to the χ^2_6 distribution (there are 2 variables at lag 2, 2 at lag 3 and 2 at lag 4).

It has a p-value equal to 0.8111, again entirely equivalent to the outcome of the F test. We do not reject the null that lags 2…4 are zero.

Model reduction, F and LR tests to choose lags

comparing the 4 lag model with the zero lag model

tests whether at least one of lags 1…4 is significant.

$\log L_U =$ -230.382

$\log L_R =$ -422.084

the test statistic is 2{-230.382 +422.084 } = 383.404

has a p-value close to 0 in χ^2_9