EC408 Topics in Applied Econometrics

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Autoregressive distributed lag (ADL) models

two stage Engle-Granger process

1)we fit the long run static model to give the residuals then test these to check that they are I(0),
2) if yes, the second stage is to fit the ECM model

Problems such as

finite sample bias in the $\hat{e}$

Solution

<u>estimate both the short run dynamic model and the long run static model jointly, using an ADL model</u>

Autoregressive distributed lag (ADL) models

‘a workhorse of the modern literature on time-series analysis’ (Greene, 2003, p.579)

$$Y_t = \gamma_0 X_t + \gamma_1 X_{t-1} + \alpha_1 Y_{t-1} + u_t \qquad (35)$$

We refer to equation (35) as an ADL(1,1) one lag for each of *X* and *Y*

we could have more variables and lags

more general specification is ADL(p,q).
This is defined by Stock and Watson(2007, p.544)

Autoregressive distributed lag (ADL) models

$$Y_t = \gamma_0 X_t + \gamma_1 X_{t-1} + \alpha_1 Y_{t-1} + u_t \qquad (35)$$

gives the same parameters as the ECM
and allows the test of cointegration

$$\Delta Y_t = b_1 \Delta X_t - b_2 (Y_{t-1} - \hat{Y}_{t-1}) + u_t$$

$$Y_t = \beta_1 X_t + e_t$$

$$\hat{Y}_{t-1} = \hat{\beta}_1 X_{t-1}$$

$$Y_{t-1} - \hat{Y}_{t-1} = \hat{e}_{t-1}$$

How the ADL and ECM are related: the long-run parameter

$$\Delta Y_t = \gamma_0 \Delta X_t - (1-\alpha_1)(Y_{t-1} - \beta_1 X_{t-1}) + u_t \qquad (37)$$

β_1 is a function of the ADL parameters, we use this later

The reason why $\beta_1 = (\gamma_0 + \gamma_1)/(1-\alpha_1)$ is because at equilibrium economic forces are in balance and there is no tendency to change, so that $Y_t = Y_{t-1}, X_t = X_{t-1}$. Substituting into the ADL gives

$$\begin{aligned} Y_t &= \gamma_0 X_t + \gamma_1 X_t + \alpha_1 Y_t + u_t \\ Y_t(1-\alpha_1) &= (\gamma_0 + \gamma_1) X_t + u_t \\ Y_t &= \frac{(\gamma_0 + \gamma_1)}{(1-\alpha_1)} X_t + e_t \\ Y_t &= \beta_1 X_t + e_t \end{aligned} \qquad (38)$$

The error term e_t differs from u_t because we have divided by $1-\alpha_1$.

From ADL to ECM

For the ECM, we can obtain this from the ADL model by noting that the parameters γ_0, γ_1 must sum to an arbitrary constant γ_2. In other words

$$\gamma_0 + \gamma_1 = \gamma_2 \qquad (39)$$

Hence

$$\beta_1 = \frac{\gamma_2}{(1-\alpha_1)}. \qquad (40)$$

$$\gamma_1 = \gamma_2 - \gamma_0$$

This means that

$$Y_t = \gamma_0 X_t + (\gamma_2 - \gamma_0) X_{t-1} + \alpha_1 Y_{t-1} + u_t$$

rearrange

$$Y_t = \gamma_0 (X_t - X_{t-1}) + \alpha_1 Y_{t-1} + \gamma_2 X_{t-1} + u_t$$

subtract Y_{t-1}

$$\Delta Y_t = \gamma_0 \Delta X_t - (1-\alpha_1) Y_{t-1} + \gamma_2 X_{t-1} + u_t$$

$$\Delta Y_t = \gamma_0 \Delta X_t - (1-\alpha_1)(Y_{t-1} - \beta_1 X_{t-1}) + u_t$$

$$\beta_1 = \frac{\gamma_2}{(1-\alpha_1)}$$

How the ADL and ECM are related

- If a cointegrating long run relationship exists, then there must be a stationary ADL and also an ECM representation in which all the variables are stationary.
- The ADL can have as many lags are as necessary to whiten the residuals *u* so that they are well behaved and simply 'noise', hence the standard t tests will be valid. With sufficiently long lags for *X* and *Y* we get more precise estimates of the long run parameter β_1.
- In contrast directly estimating the cointegrating regression $Y_t = \beta_1 X_t + e_t$ is problematic because $\hat{\beta}_1$ is biased in small samples and t tests are not valid with I(1) variables.

From ADL(1,1) to ADL(p,q)

ADL(1,q)

$$Y_t = \alpha_1 Y_{t-1} + \gamma_0 X_t + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + ... + \gamma_q X_{t-q} + u_t \qquad (42)$$

ADL(p,q)

$$Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} ... + \alpha_p Y_{t-p} + \gamma_0 X_t + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + ... + \gamma_q X_{t-q} + u_t \qquad (44)$$

The notation is becoming cumbersome, so let us simplify. It is useful to use a lag operator L (Stock and Watson, 2007, p. 634) to do this. Hence

$$L^r Y_t = Y_{t-r}$$
$$\alpha(L) = \sum_{i=1}^{p} \alpha_i L^i = \alpha_1 L^1 + \alpha_2 L^2 + \alpha_3 L^3 + ... + \alpha_p L^p \tag{46}$$

Combining these gives

$$\alpha(L)Y_t = \sum_{i=1}^{p} \alpha_i L^i Y_t = \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} + ... + \alpha_p Y_{t-p} \tag{47}$$

So the ADL(p,q) becomes

$$(1-\alpha L)Y_t = \gamma(L) X_t + u_t \tag{48}$$

Or

$$Y_t - \sum_{i=1}^{p} \alpha_i Y_{t-i} = \sum_{i=0}^{q} \gamma_i X_{t-1} + u_t \tag{49}$$

Additional variables in the ADL

variables ($X_1, X_2, ..., X_{g-1}$).

Rather than $(1-\alpha L)Y_t = \gamma(L)X_t + u_t$ for just one *X* and one *Y*,

we now have

$$(1-\alpha L)Y_t = b_1(L)X_{1t} + b_2(L)X_{2t} + ... + b_{g-1}(L)X_{g-1t} + u_t \quad (53)$$

With each $b_i, i = 1, ..., g-1$, comprising a vector of parameters appropriate to each

$X_i, i = 1, ..., g-1$.

Additional variables in the ADL

we do reject Ho: $\delta = 0$ in favour of $\delta < 0$ for INFLAT, since ADF-INFLAT = -6.197** Critical values used in ADF test: 5%=-3.439, 1%=-4.019.

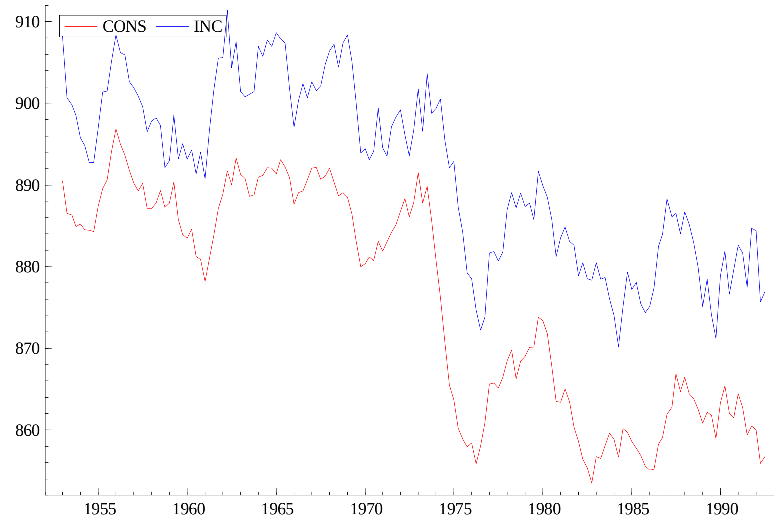

Fig 10 variables CONS and INC, both ~ I(1)

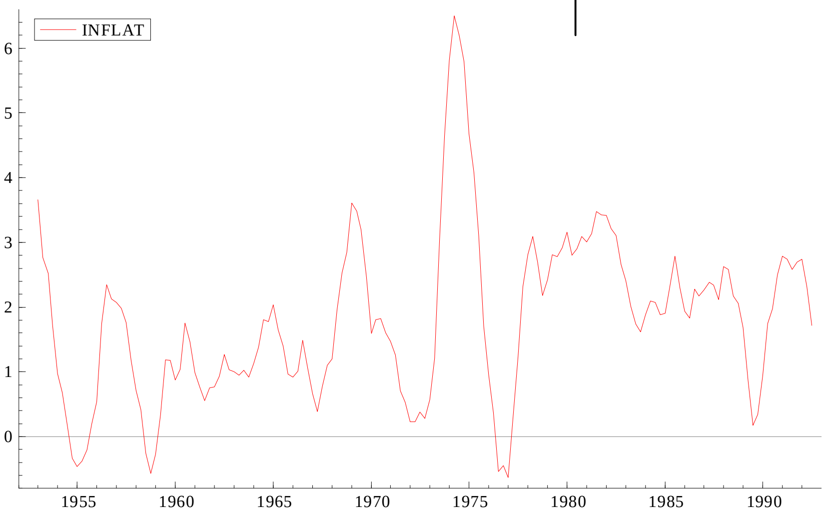

Fig 11 Variable INFLAT ~ I(0)

ADF-CONS = -3.034 which indicates that Ho: $\delta = 0$ should not be rejected in favour of $\delta < 0$ using critical values : 5%=-3.439, 1%=-4.019, We also find that $\delta = 0$ is not rejected for INC, since ADF-INC = -3.14 Critical values used in ADF test: 5%=-3.439, 1%=-4.019.

Additional variables in the ADL

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EQ(95) Modelling CONS by OLS (using DATA.IN7)
       The estimation sample is: 1953 (2) to 1992 (3)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
CONS_1               0.830024    0.02401     34.6   0.000   0.8865
INC                  0.479933    0.02759     17.4   0.000   0.6642
INC_1               -0.311087    0.03543    -8.78   0.000   0.3351
INFLAT              -0.764010     0.1863    -4.10   0.000   0.0990
INFLAT_1            -0.230348     0.2060    -1.12   0.265   0.0081

sigma                 1.09463  RSS                 183.325319
log-likelihood       -235.937  DW                        1.95
no. of observations       158  no. of parameters            5
mean(CONS)            875.848  var(CONS)              181.971

CONS =  + 0.83*CONS_1 + 0.4799*INC - 0.3111*INC_1 - 0.764*INFLAT
(SE)     (0.024)       (0.0276)     (0.0354)       (0.186)
        - 0.2303*INFLAT_1
         (0.206)
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