EC408 Topics in Applied Econometrics

B Fingleton, Dept of Economics, Strathclyde University

From ADL models to VAR models

Autoregressive Distributed Lag models

Our most general model thus far is

$$(1-\alpha L)Y_t = b_1(L)X_{1t} + b_2(L)X_{2t} + \ldots + b_{g-1}(L)X_{g-1t} + u_t \qquad (66)$$

And with g-1 = 2 right hand side variables and one lag this reduces to

$$Y_t = \alpha_1 Y_{t-1} + b_{10}X_{1t} + b_{11}X_{1t-1} + b_{20}X_{2t} + b_{21}X_{2t-1} + u_t \qquad (67)$$

Additional variables in the ADL

we do reject Ho: $\delta = 0$ in favour of $\delta < 0$ for INFLAT, since ADF-INFLAT = -6.197** Critical values used in ADF test: 5%=-3.439, 1%=-4.019.

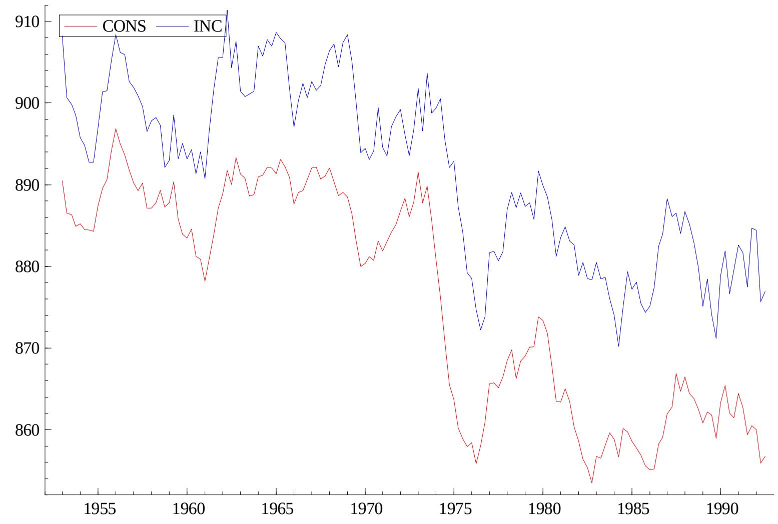

Fig 10 variables CONS and INC, both ~ I(1)

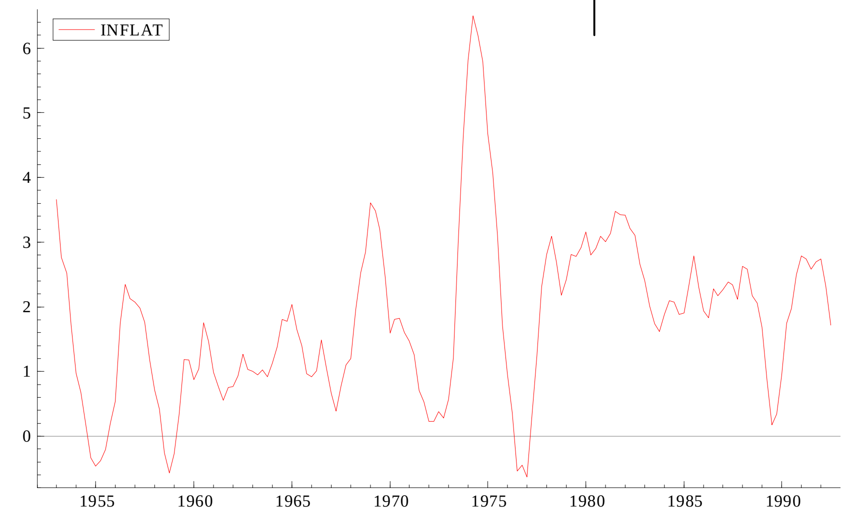

Fig 11 Variable INFLAT ~ I(0)

ADF-CONS = -3.034 which indicates that Ho: $\delta = 0$ should not be rejected in favour of $\delta < 0$ using critical values : 5%=-3.439, 1%=-4.019, We also find that $\delta = 0$ is not rejected for INC, since ADF-INC = -3.14 Critical values used in ADF test: 5%=-3.439, 1%=-4.019.

ADL as an ECM

Empirical example g = three variables (Y, X_1, X_2) and a cointegrating relationship between them.

A linear combination of two I(1) variables (CONS, INC) and an I(0) variable (INFLAT) be combined to give an I(0) variable

$$\hat{e}_{1t} = Y_t - \hat{\beta}_{11} X_{1t} - \hat{\beta}_{21} X_{2t} \sim \text{I}(0).$$

This can also be written as

$$\hat{e}_{1t} = \hat{\beta}_1' \text{Y}_t$$

$$\hat{\beta}_1 = \begin{bmatrix} 1 \\ -\hat{\beta}_{11} \\ -\hat{\beta}_{21} \end{bmatrix} \quad (68)$$

$$\text{Y}_t = \begin{bmatrix} Y_t \\ X_{1t} \\ X_{2t} \end{bmatrix}$$

ADL as an ECM

$$Y_t = \alpha_1 Y_{t-1} + b_{10} X_{1t} + b_{11} X_{1t-1} + b_{20} X_{2t} + b_{21} X_{2t-1} + u_t$$

There is a stationary ADL and also an equivalent ECM representation in which all the variables are stationary. We can avoid problems of spurious regression since the model

$$\begin{aligned}
\Delta Y_t &= \gamma_0 \Delta X_{1t} + \gamma_1 \Delta X_{2t} - (1-\alpha_1)(Y_{t-1} - \hat{\beta}_{11} X_{1t-1} - \hat{\beta}_{21} X_{2t-1}) + u_t \\
\Delta Y_t &= \gamma_0 \Delta X_{1t} + \gamma_1 \Delta X_{2t} - (1-\alpha_1)\hat{e}_{1t-1} + u_t \\
\Delta Y_t &= \gamma_0 \Delta X_{1t} + \gamma_1 \Delta X_{2t} - (1-\alpha_1)\beta_1' \mathrm{Y}_{t-1} + u_t
\end{aligned} \tag{69}$$

comprises stationary variables

VAR models

Vector Autoregressive Models
Fundamental idea : we have multiple endogenous variables

- In order to properly appreciate the ideas here, we need to conceive of our model of multiple I(1) variables that may be cointegrated as a VAR (vector autoregression).
- There are many advantages in treating systems as VARs, as set out for example by Greene(2004, p. 587).
- One notable advantage is that we can have multiple endogenous variables.
- Stock and Watson(2007, p. 638) set out the basic VAR involving only endogenous variables (X,Y).

VAR models

- With just 2 variables *X*, and *Y*, assume we wish to forecast both variables not by two separate models but as part of a single model.
- The specification with p lags is

$$
\begin{aligned}
Y_t &= b_{11}Y_{t-1} + \ldots + b_{1p}Y_{t-p} + c_{11}X_{t-1} + \ldots + c_{1p}X_{t-p} + u_{1t} \\
X_t &= b_{21}Y_{t-1} + \ldots + b_{2p}Y_{t-p} + c_{21}X_{t-1} + \ldots + c_{2p}X_{t-p} + u_{2t}
\end{aligned}
\tag{70}
$$

Where b, c are unknown coefficients and u_1, u_2 are errors. These models can be estimated by OLS, provided all the standard regression assumptions are satisfied.

VECMs

Vector error correction models

- we know that there are severe problems for regression, leading to spurious regression, when variables possess stochastic trends.
- In other words in the VAR, as in single equation models, the variables may be I(1). However we have acknowledged the possibility of the I(1) variables being cointegrated, thus allowing a regression involving stationary variables.

Vector error correction models

In this case we a have a vector error correction model (c.f Stock and Watrson 2007, p. 656, Davidson and Mackinnon, 2004, p.629-630)

$$\Delta Y_t = b_{11}\Delta Y_{t-1} + \ldots + b_{1p}\Delta Y_{t-p} + c_{11}\Delta X_{t-1} + \ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + c_{1p}\Delta X_{t-p} - (1-\alpha_1)(Y_{t-1} - \beta_1 X_{t-1}) + u_{1t}$$
$$\Delta X_t = b_{21}\Delta Y_{t-1} + \ldots + b_{2p}\Delta Y_{t-p} + c_{21}\Delta X_{t-1} + \ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + c_{2p}\Delta X_{t-p} - (1-\alpha_2)(Y_{t-1} - \beta_1 X_{t-1}) + u_{2t} \quad (71)$$

So just as with the single equation error correction model, we have the stationary differences model augmented by the error correction term $(Y_{t-1} - \beta_1 X_{t-1})$ which, despite both X and Y being I(1), is I(0). If this is not the case, and the error correction term remains I(1), then this will produce, for instance, poor out-of-sample forecast performance for $\Delta Y_t, \Delta X_t$.