Rumfitt's Theory of Predication*

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1 Introduction

In the second chapter of his book *Frege: Philosophy of Language* [4] Dummett explains how Frege solved the problem of multiple generality. I will briefly summarise the main idea behind Frege's solution. Consider the following three sentences:

(A) Jack loves everybody.

(B) Somebody loves everybody.

(C) Somebody loves Jill.

Before Frege there was no satisfactory way of explaining both the validity of the inference of (B) from (A) and also that of the inference of (C) from (B). Frege's solution has several components, but one key element is his ability to decompose (B) in at least two different ways. The unasserted proposition contained in (B) can be seen as obtained by attaching the expression of generality 'somebody' to the incomplete expression 'ξ loves everybody', but it can also be obtained by attaching the expression of generality 'everybody' to the incomplete expression 'somebody loves ξ'. The inference of (B) from (A) can now be seen as an instance of existential generalisation and the inference of (C) from (B) can now be seen to be an instance of universal instantiation.

Dummett gives an interpretation of incomplete expressions as being patterns, but in a fairly recent paper Rumfitt [13] has championed Geach's interpretation of incomplete expressions as being linguistic functions. Unfortunately, Rumfitt's account has several flaws. The most notable of these is that his theory cannot account for the validity either of the inference of (B) from (A) or of that of (C) from (B). I agree with Rumfitt

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that incomplete expressions are best thought of as linguistic functions, but the flaws in his presentation of the function-interpretation may give that interpretation a bad name. So, after giving an exposition of Rumfitt's theory of predication and showing why it cannot solve the problem of multiple generality, I explain how the function-interpretation can be rehabilitated to explain the validity of inferences involving several expressions of generality. After that I present three further flaws in Rumfitt's theory of predication.

2 Exposition

Rumfitt's discussion makes extensive use of Geach's notion of a linguistic function [8, pp. 143–144]. An n-place linguistic function, for $n > 0$, can be defined recursively as follows:

> An *n-place linguistic function* is a function which takes n arguments, each of which is either a linguistic expression or a linguistic function, and returns as its value either a linguistic expression or a linguistic function.

Within the collection of linguistic functions Rumfitt distinguishes between transparent and non-transparent ones (603).[1] I take what Rumfitt calls a *transparent* linguistic function to be one which has the property that every one of its argument-places is a transparent context. Rumfitt is concerned to isolate the collection of first-level predicables within the set of all transparent linguistic functions which yield a proposition when applied to a finite number of singular terms.[2] (When there is no danger of confusion, I shall simply use the word 'predicable' instead of the expression 'first-level predicable'.) Rumfitt cannot simply say that all such functions are predicables, because he constructs a transparent linguistic function which is clearly not a predicable (603). This presents him with the problem of distinguishing between predicables and rogue linguistic functions.[3] He does this by means of a recursive definition which aims to show how all predicables can be constructed by a small number of operations from a class of basic predicables (604–605). This definition makes use of various operators. Three of these are Ref, Ins and Alt:

> Ref(ϕ) 'maps an arbitrary two-place predicable $h(\xi, \eta)$ to the one-place $f(\xi)$ in such a way that for every name $\mathbf{n}$, $f(\mathbf{n}) = h(\mathbf{n}, \mathbf{n})$' (604).

[1] When a page number occurs on its own in what follows it is to be taken as referring to the appropriate page of Rumfitt's article.

[2] Rumfitt uses the terms 'predicable' and 'proposition' in the same way that Geach does. For Geach, a predicable is what other logicians call a 'predicate'. Geach reserves the term 'predicate' for a predicable that is actually used to make a predication. Thus, the predicable 'ξ smoked a pipe' occurs in both of the propositions 'A Cambridge philosopher smoked a pipe and drank a lot of whisky' and 'Russell smoked a pipe', but it is only a predicate in the second example [9, pp. 110–112]. For Geach, a proposition is a linguistic expression which can be used to express a complete thought. It is not some strange non-linguistic entity [10, p. 139].

[3] What Rumfitt refers to as a 'rogue linguistic function' I have elsewhere [2, p. 94] called a 'pathological linguistic function'.

> Ins$[\mathbf{n}, i](\phi)$, where i is either 1 or 2, is that higher-order function which puts the name $\mathbf{n}$ into the ith argument-place of the two-place predicable ϕ.
>
> Alt is that two-place propositional function which forms the disjunction of its two propositional arguments.

Although Alt is the only example of a two-place propositional function that Rumfitt gives, it is easy to see how further such functions could be introduced.

It is now possible for me to present my understanding of Rumfitt's characterisation of what a predicable is. I do this in the form of a recursive definition:

> A *predicable* is either a basic predicable or it is obtained by combining two predicables by means of a two-place propositional function or it is obtained by applying either Ref or Ins to a predicable (or an analogue of these for predicables of higher arity).

Rumfitt does not explicitly consider negation as a linguistic function, but it is easy to see how it could be incorporated into this definition.

To complete this definition it is only necessary to specify what is to count as a basic predicable. In effect, Dummett's simple predicates and simple relational signs are Rumfitt's basic predicables (604, fn. 6).[4] It is true that Dummett does not think of these as linguistic functions, but Rumfitt's intention is clear. If 'snores', say, is a simple predicate for Dummett and 'loves' a simple relational sign, then 'ξ snores' and 'ξ loves η' are basic predicables for Rumfitt. Rumfitt does not distinguish between simple and complex expressions in the way that Dummett does and all his predicables are incomplete expressions.

Rumfitt is well aware that one and the same simple proposition can be decomposed in several different ways (602).[5] To use his example, 'Hegel contradicts Hegel' is the result of applying the linguistic function 'Hegel contradicts ξ' to 'Hegel', but it is also the result of applying 'ξ contradicts η' to an ordered pair of names both of which are 'Hegel'. There are also other possible decompositions. Out of all such possible decompositions Rumfitt singles out one to be what he calls the *fundamental decomposition* (604) or *analysis* (610) of the proposition. This is the decomposition in which the linguistic function involved is a basic predicable (604). Although Rumfitt's discussion of decomposition only applies to simple propositions, its extension to propositions involving propositional functions and quantifiers is straightforward. For example, although the sentence 'Hegel contradicts somebody or everybody contradicts Hegel' has many decompositions, the fundamental one is that in which it is first broken down into the two sentences 'Hegel contradicts somebody' and 'everybody contradicts Hegel' and the propositional function Alt. The sentence 'Hegel contradicts somebody' is then broken down into 'Hegel contradicts ζ' and 'somebody'. The linguistic function 'Hegel contradicts ζ' is then broken down into Ins['Hegel', 1] and 'ξ contradicts ζ'. Similarly, 'everybody contradicts Hegel' is broken down into 'ξ contradicts Hegel' and

[4] Dummett's account of simple and complex predicates and relational signs occurs in [4, 27-33].

[5] By a *simple* proposition he means one in which neither propositional connectives nor quantifiers occur (605).

'everybody' and the linguistic function 'ξ contradicts Hegel' is then broken down into Ins['Hegel', 2] and 'ξ contradicts ζ'.

3 Criticism

3.1 The Extraction-procedure and Multiple Generality

One of the distinctive features of Frege's account of incomplete expressions is that he allows such expressions to be formed by omitting one or more occurrences of an expression from another expression. I call this operation the *extraction-procedure*.[6] In the context of an account of predication Frege would allow a one-place predicable to be formed by the omission of one or more occurrences of a singular term from a proposition. Similarly, an n-place predicable, for any positive integer n, can be obtained by removing one or more occurrences of each of n singular terms from a proposition. Because Rumfitt is mainly concerned with issues relating to predicables, in what follows when I use the expression 'extraction-procedure' I will mean this restricted version of it. The extraction-procedure allows us, for example, to remove the proper name 'Jack' from the proposition 'Jack loves everybody' in order to form the incomplete expression 'ξ loves everybody'. An analogue of it also plays an important role in Dummett's notion of the step-by-step construction of sentences [4, pp. 16 and 23], on which Rumfitt claims to base his own account (604, fn. 6), though Dummett [4, p. 29] only considers the cases in which it is used to form one-place or two-place predicables.[7] The absence of the extraction-procedure from Rumfitt's account of the formation of predicables means that he cannot, for example, explain how the conclusion 'Somebody loves everybody' follows logically from the premise 'Jack loves everybody'. This is because his formation rules do not allow us to construct the incomplete expression 'ξ loves everybody'. The account of the quantifiers that he gives (606) only explains how they can be used to form propositions by being applied to *one-place* predicables. He has no procedure which allows a quantifier to combine with an n-place predicable, where $n > 1$, in order to form an $(n-1)$-place predicable.

Rumfitt's account of predication could not be patched up simply by adding the extraction-procedure to it. This is because it is important for him to be able to single out the *unique* analysis of a proposition from all the different ways in which that proposition can be decomposed. He needs to be able to do this, for example, in order to explain what it is for propositions to be equipollent. In his discussion of equipollence the notion of a proposition's analysis figures essentially (612).

Why would adding the extraction-procedure to Rumfitt's account of the formation of predicables destroy the possibility of singling out the unique analysis of a propo-

[6]The word 'extraction' is borrowed from Dummett who formulates 'a principle of the extraction of functions' [5, p. 281] in the course of defending his exegesis of Frege. A version of this operation occurs in §30 of *Grundgesetze* [7] where it is called the 'second procedure for forming names of first-level functions'.

[7]The reason why I attribute to Dummett an *analogue* of the extraction-procedure is because, as I have explained it, the result of using this operation is a linguistic function, whereas in Dummett's version the result is a pattern.

sition? This is because a predicable like 'ξ loves everybody' can be obtained in an unlimited number of ways. For example, it can be obtained by removing 'Jack' from 'Jack loves everybody' or by removing 'Jill' from 'Jill loves everybody' or by removing 'Maxine' from 'Maxine loves everybody' and so on. None of these ways of obtaining 'ξ loves everybody' has any claim of priority over any of the others. This means, for example, that a proposition like 'Somebody loves everybody' has an unlimited number of decompositions none of which is more fundamental than any of the others. This can be seen clearly by considering two possible decompositions:

$$
\begin{tikzcd}
\text{`Somebody loves everybody'} \\
\text{`ξ loves everybody'} \arrow[u, "E(\phi)"'] \\
\text{`Jack loves everybody'} \arrow[u, "\text{extraction-procedure}"'] \\
\text{`Jack loves ζ'} \arrow[u, "U(\phi)"'] \\
\text{`$\xi$ loves $\zeta$';} \arrow[u, "\text{Ins[`Jack'}, 1]"']
\end{tikzcd}
\qquad
\begin{tikzcd}
\text{`Somebody loves everybody'} \\
\text{`ξ loves everybody'} \arrow[u, "E(\phi)"'] \\
\text{`Jill loves everybody'} \arrow[u, "\text{extraction-procedure}"'] \\
\text{`Jill loves ζ'} \arrow[u, "U(\phi)"'] \\
\text{`$\xi$ loves $\zeta$'.} \arrow[u, "\text{Ins[`Jill'}, 1]"']
\end{tikzcd}
$$

Here, $U(\phi)$ and $E(\phi)$ are both second-level linguistic functions (606). $U(\phi)$ attaches 'everybody' to a one-place predicable ϕ and $E(\phi)$ attaches 'somebody' to a one-place predicable ϕ. Both of the above decompositions involve the predicable 'ξ loves ζ'. To obtain 'somebody loves everybody' from this we first have to insert a name into its first argument-place. In the case of the decomposition on the left this name is 'Jack'. Then we attach the quantifier 'everybody' to this. Using the extraction-procedure this is transformed into 'ξ loves everybody' to which 'somebody' can then be attached to obtain 'somebody loves everybody'. The decomposition on the right is similar except that the name 'Jill' is used instead of 'Jack'. Although both decompositions terminate in the same predicable they are nevertheless distinct and because they are different 'somebody loves everybody' does not have a unique decomposition. In fact, the predicable 'ξ loves ζ' occurs in an unlimited number of decompositions. For example, it figures in the decompositions of at least the following propositions: 'If no one loves Jack, then Maxine does not', 'Either everybody loves Jill or Jack loves Maxine', 'Somebody loves somebody' and 'Everybody loves everybody'.

As already mentioned, I am sympathetic to the function-interpretation of incomplete expressions and elsewhere [2] I consider a problem which is more general than the one that Rumfitt considers. I am there interested in isolating the collection of all Fregean incomplete expressions within the class of all linguistic functions. I devise various pathological linguistic functions which show that not all linguistic functions are incomplete expressions. The criterion which I propose for distinguishing between Fregean incomplete expressions and pathological linguistic functions is the following one [2, p. 94]:

> an unsaturated expression is a linguistic function that can be represented by means of Frege's xi-notation (or an extension of this notation in order to cope with functions of higher-level or greater arity) and whose referent is an entity which is of a type that occurs somewhere in the Fregean hierarchy of types and which is not a basic type.[8]

Having isolated the class of all incomplete or unsaturated expressions it is easy to locate the collection of all predicables within this class. A predicable is an unsaturated expression which yields a proposition when applied to a finite number of singular terms. Under this characterisation of predicables both 'ξ loves everybody' and 'somebody loves ξ' turn out to be predicables and so we can explain the validity of the inference of (B) from (A) and of that of (C) from (B). Furthermore, this way of characterising the collection of all predicables is preferable to Rumfitt's because it is compatible with the use of the extraction-procedure.

Rumfitt's theory of predication has a number of further flaws and it is to these that I now turn my attention.

3.2 Predication and Assertion

As Geach has pointed out Frege 'demonstrated the need to make an absolute distinction between predication and assertion' [9, p. 253]. Rumfitt joins together what Frege separated. He considers predicables to be linguistic functions whose values are *asserted* propositions. That this is so can be seen by considering the chain of reasoning which he presents on p. 600:

(1) Everybody contradicts himself.
So, (2) Hegel contradicts Hegel.
So, (3) Somebody contradicts Hegel.

For Rumfitt, each link in this is an asserted proposition. He talks, for example, about what (3) 'affirms' (600). Further on he says that it is expedient to view the second link in this chain, namely 'Hegel contradicts Hegel', first as the value of the linguistic function 'ξ contradicts ξ' for the argument 'Hegel' and second as the value of the linguistic function 'ξ contradicts Hegel' for the same argument (605). He considers both of these linguistic functions to be predicables. As I have shown elsewhere [2,

[8] In quoting this passage I have replaced the word 'polyadicity' by 'arity', which I now prefer.

pp. 93–95] a linguistic function whose value is an asserted proposition is pathological in the sense that it is not a Fregean incomplete expression. Using the symbol '$\vdash$' as an explicit assertoric-force indicator this point can be illustrated by noting that '$\vdash \xi$ snores' is not an incomplete expression for Frege (though it is for Rumfitt). That Frege would not consider something like '$\vdash \xi$ snores' to be an incomplete expression is brought out very clearly in the footnote on p. 22 of *Funktion und Begriff* [6]. (I quote and discuss this footnote extensively elsewhere [2, section 4].)

3.3 Combination Problems

For Rumfitt the propositional functor Alt is a linguistic function which makes a proposition out of two other propositions (605). Thus, when applied to 'Jack snores' and 'Jill hallucinates' it yields 'Either Jack snores or Jill hallucinates'. Rumfitt, however, also allows Alt to combine predicables (605). This means that he would allow the value of Alt, when applied to 'ξ snores' and 'η hallucinates', to be 'Either ξ snores or η hallucinates'. The problem with this is that the collection of ordered pairs of predicables and the domain of the linguistic function Alt are disjoint. This can be expressed in more Fregean language by saying that predicables do not fit into the argument-places of Alt. This difficulty is one example of a class of problems that Potts has called *combination problems* [11, pp. 12–18]. Rumfitt's account of the formation of predicables is incomplete because he does not give any account of how he intends to solve such problems. The solution of combination problems is no easy or straightforward matter. Potts, for one, has made several attempts to solve them. His recent book contains a thorough discussion of the problems and also his latest solutions of them [12, section 2.5]. Rumfitt must be aware of the existence of combination problems as he reviewed Potts's book [14]!

3.4 Opaque Contexts

Rumfitt not only elaborates Frege's theory of predication, he also presents several applications of it. One of these relates to certain kinds of attribution (612–622). I do not intend to discuss every nuance of Rumfitt's discussion here. I only want to look at the role that his theory of predication has in it. What is problematic about this is that although Rumfitt elaborates a theory that only deals with transparent predicables he applies this theory to predicables some of whose argument-places are opaque. His mistake is analogous to that which would be made by a mathematician who developed a theory of Euclidean geometry and then tried to use that theory to reason about the properties of very large triangles on the surface of the Earth. Not surprisingly, in both cases, contradictions appear. What is needed, in both cases, is a different theory. In the mathematical case a theory of spherical geometry is required, whereas in the case in hand what is needed is a theory of predication that can handle both transparent predicables and those with one or more opaque argument-places. It is true that Rumfitt subscribes to a version of semantic innocence and he champions the paratactic theory, but in some of the reasoning that he presents as supporting these positions he discusses expressions like 'Jill says that ξ snores' and it is this discussion that I am concerned with here. He applies his theory to such expressions when that theory was not devised to deal with them. It should be noted that Rumfitt's discussion of these expressions is logically prior to his elaboration of the paratactic theory and not dependent on it. I will now bring out some of the specific difficulties in Rumfitt's attempt to apply his theory of predication to predicables with one or more opaque argument-places.

Rumfitt introduces the linguistic function $f(\xi)$ as follows (613):

> $f(\xi)$ is that linguistic function that maps an arbitrary name **n** to the proposition $\ulcorner A$ says that **n** is $F\urcorner$ ... .

He assumes that there exist proper names **a** and **b** which are such that $f(\mathbf{a})$ is true, $f(\mathbf{b})$ is false and yet **a** and **b** have the same bearer. He then goes on to argue as follows:

> Accordingly, by the basic principle of decomposition, formulated ... as (C), the object that is the denotation of **a** falls under the concept symbolized by $f(\xi)$, while the denotation of **b** does not. But since *ex hypothesi* the denotation of **a** is identical with the denotation of **b**, it follows from this that one and the same object both falls under and fails to fall under one and the same concept—which, of course, is a contradiction.

What (C), stated on p. 602, amounts to in this context is that the proposition $f(\mathbf{n})$ is true iff the denotation of the name **n** falls under the concept symbolised by the predicable $f(\xi)$. Because I am going to talk about two different contradictions, I will refer to the one that Rumfitt discusses as the *explicit* contradiction. The other one, to be introduced below, will be called the *implicit* contradiction.

There is at least one hidden assumption in Rumfitt's reasoning that he fails to notice and that is the supposition that $f(\xi)$ is a predicable. In fact, $f(\xi)$ is not a predicable because it fails to meet at least one of the criteria for something to be a predicable that Rumfitt lays down. As already mentioned, Rumfitt defines the collection of predicables to be a proper subset of the collection of *transparent* linguistic functions which yield a proposition when applied to a finite number of singular terms, but in his discussion of $f(\xi)$ he has assumed that it is not transparent. That means that it cannot be a predicable. In other words, $f(\xi)$ is not a Fregean incomplete expression. Furthermore, as a concept for Frege is the referent of an incomplete expression which makes a proposition out of a singular term, the linguistic function $f(\xi)$ does not refer to a concept. What this discussion has established is that Rumfitt implicitly contradicts himself by asserting that $f(\xi)$ is a predicable when, according to his own definition, it is not one. I will refer to this contradiction as the *implicit* one. If Rumfitt were to resolve this implicit contradiction by consistently denying that $f(\xi)$ is a predicable (and making suitable changes to ensure this), then the derivation of the explicit contradiction in the piece of reasoning quoted above could not be carried out because his basic principle of decomposition would not apply to $f(\xi)$.

Rumfitt sees the source of the explicit contradiction to be the hidden assumption that $\ulcorner A$ says that $p\urcorner$ is a unitary proposition (615). He offers a paratactic account of propositions like ⌜A says that p⌝, ⌜A believes that p⌝, ⌜A hopes that p⌝ and so on. This involves analysing such propositions into a pair of propositions. As a predicable is defined to be a linguistic function whose value is a unitary proposition, anything whose value is a pair of propositions cannot be a predicable (615) and so the explicit contradiction cannot arise. This way of resolving the explicit contradiction has at least one major defect. No matter how a proposition like

(D) Jill says that Jack snores.

is analysed it still remains a unitary proposition. Just because (D) is analysed into the pair of propositions

(E) Jill says that. Jack snores.

it does not follow that (D) is not unitary. Rumfitt argues (614–615) that a proposition like (D) ceases to exist because in the paratactic theory he favours it is analysed into the pair of propositions (E). This cannot be correct. In order to propose an analysis of (D) into (E) we have to first understand (D) independently of knowing that it can be analysed into (E). The proposition (D) is not introduced into our language as the *definiendum* of an abbreviatory definition whose *definiens* is (E). In order to understand (D) we have to understand its constituents and how they are put together. Although (D) can be seen as put together in many ways, the last step in one of its constructional histories sees it as being the result of applying 'Jill says that ξ snores' to 'Jack' and this is a different linguistic function from the one that maps 'Jack' to (E). Rumfitt's account of linguistic functions like 'Jill says that ξ snores' is, thus, in need of improvement.

4 Conclusion

Frege put forward a revolutionary theory of predication which he used to solve the traditional problem of multiple generality. His theory makes use of what he called *incomplete expressions* or *unsaturated expressions*. At least two different interpretations of these have been put forward. Incomplete expressions can be thought of as being either patterns or linguistic functions. Rumfitt favours the function-interpretation and so do I. Unfortunately, Rumfitt's presentation of this has a number of flaws which I have drawn attention to. Its most significant limitation is that it cannot account for the validity of inferences involving more than one expression of generality. The function-interpretation can, however, be rehabilitated in order to overcome this failing, as I have shown. The version of the function-interpretation that I favour also has other uses. It allows us to provide a coherent account of Fregean incomplete expressions on the linguistic level [2], in the realm of sense [3] and also in the realm of reference [1]. Rumfitt's work is valuable, however, because it makes clear that the full potential of Frege's theory of predication has still not been realised and that it is likely to give rise to further developments in the philosophy of language.

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