$k$-Zero-Divisor Hypergraphs

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Algebraic structures + Graph structures
Algebraic structures + Graph structures = \(k\)-Zero-Divisor
Algebraic structures

Graph structures

$k$-Zero-Divisor

Hypergraph
Algebraic structures

$k$-Zero-Divisor

Hypergraph

$k$-Zero-divisor hypergraphs
Algebraic structures
Algebraic structures

A nonzero nonunit element $z_1$ is said to be $k$-zero-divisor
A nonzero nonunit element $z_1$ is said to be $k$-zero-divisor if there exist $k - 1$ distinct nonunit elements $z_2, z_3, z_4, \ldots, z_k$ differ from $z_1$ such that
A nonzero nonunit element $z_1$ is said to be a $k$-zero-divisor if there exist $k - 1$ distinct nonunit elements $z_2, z_3, z_4, \ldots, z_k$ differ from $z_1$ such that

$z_1 z_2 z_3 \cdots z_k = 0$

and the products of elements of any $k - 1$ subset of $\{z_1, z_2, z_3, \ldots, z_k\}$ are nonzero.

Chelvam et al.
Example of $k$-zero-divisor
Example of $k$-zero-divisor

Consider $\mathbb{Z}_{30}$
Example of $k$-zero-divisor

Consider $\mathbb{Z}_{30}$

We know that $\overline{2} \cdot \overline{3} \cdot \overline{5} = 0$
Example of $k$-zero-divisor

Consider $\mathbb{Z}_{30}$

We know that $\overline{2} \cdot \overline{3} \cdot \overline{5} = \overline{0}$

$\overline{2} \cdot \overline{3} \neq \overline{0}$, $\overline{2} \cdot \overline{5} \neq \overline{0}$, $\overline{3} \cdot \overline{5} \neq \overline{0}$
Example of \( k \)-zero-divisor

Consider \( \mathbb{Z}_{30} \)

We know that \( 2 \cdot 3 \cdot 5 = 0 \)

\( 2 \cdot 3 \neq 0, \quad 2 \cdot 5 \neq 0, \quad 3 \cdot 5 \neq 0 \)

We obtain that \( \bar{2} \) is a 3-zero-divisor
Graph structures
Graph structures

Hypergraph $\mathcal{H}(V, \mathcal{E})$ or $\mathcal{H}$
Graph structures

Hypergraph $\mathcal{H}(V, \mathcal{E})$ or $\mathcal{H}$

- $V$ or $V(\mathcal{H})$ is a nonempty finite set of vertices or vertex set
- $\mathcal{E}$ or $\mathcal{E}(\mathcal{H})$ is a family of subsets of $V$, called set of (hyper)edges or edge set
Graph structures

Hypergraph $\mathcal{H}(V, \mathcal{E})$ or $\mathcal{H}$

- $V$ or $V(\mathcal{H})$ is a nonempty finite set of *vertices* or *vertex set*
- $\mathcal{E}$ or $\mathcal{E}(\mathcal{H})$ is a family of subsets of $V$, called set of (hyper)*edges* or *edge set*

- If each edge of $\mathcal{H}$ has size $l$, we call $\mathcal{H}$ an *$l$-uniform hypergraph*. 
Example of hypergraphs
Example of hypergraphs

3-uniform hypergraph
Complete $k$-uniform hypergraph on $n$ vertices
Complete $k$-uniform hypergraph on $n$ vertices

- $\mathcal{H}$ has all $k$-subsets of the $n$-set of vertices as edge
Complete $k$-uniform hypergraph on $n$ vertices

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Complete 3-uniform hypergraph on 4 vertices $\{1, 2, 3, 4\}$
Complete $k$-uniform hypergraph on $n$ vertices

- $\mathcal{H}$ has all $k$-subsets of the $n$-set of vertices as edges.

Complete 3-uniform hypergraph on 4 vertices $\{1, 2, 3, 4\}$

$e_1 = \{1, 2, 3\}$  $e_2 = \{1, 2, 4\}$  $e_3 = \{1, 3, 4\}$  $e_4 = \{2, 3, 4\}$
Complete $k$-uniform hypergraph on $n$ vertices

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Complete \( k \)-uniform hypergraph on \( n \) vertices

\( \mathcal{H} \) has all \( k \)-subsets of the \( n \)-set of vertices as edge

Complete 3-uniform hypergraph on 4 vertices \( \{1, 2, 3, 4\} \)

\[ e_1 = \{1, 2, 3\} \]
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Complete $k$-uniform hypergraph on $n$ vertices

- $\mathcal{H}$ has all $k$-subsets of the $n$-set of vertices as edge

Complete 3-uniform hypergraph on 4 vertices $\{1, 2, 3, 4\}$

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- $e_2 = \{1, 2, 4\}$
- $e_3 = \{1, 3, 4\}$
- $e_4 = \{2, 3, 4\}$
$k$-Partite $k$-uniform hypergraph
$k$-Partite $k$-uniform hypergraph

- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, \ldots, V_k$
- Edge set $\mathcal{E} = \{\{v_1, v_2, v_3, \ldots, v_k\}|v_j \in V_j \text{ for all } 1 \leq j \leq k\}$
\section*{$k$-Partite $k$-uniform hypergraph}

- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, \ldots, V_k$
- Edge set $E = \{\{v_1, v_2, v_3, \ldots, v_k\} | v_j \in V_j \text{ for all } 1 \leq j \leq k\}$

- Complete if $V_j = \{v_j^1, v_j^2, v_j^3, \ldots, v_j^{\left|V_j\right|}\}$ for all $1 \leq j \leq k$ and $E = \left\{\{v_1^{i_1}, v_2^{i_2}, v_3^{i_3}, \ldots, v_k^{i_k}\} | v_j^{i_j} \in V_j \text{ for all } 1 \leq j \leq k \text{ and } 1 \leq i_j \leq \left|V_j\right|\right\}$

Kuhl and Schroeder
**k-Partite k-uniform hypergraph**

- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, ..., V_k$
- Edge set $\mathcal{E} = \{\{v_1, v_2, v_3, ..., v_k\}| v_j \in V_j \text{ for all } 1 \leq j \leq k\}$

Complete if $V_j = \{v_j^1, v_j^2, v_j^3, ..., v_j^{|V_j|}\}$ for all $1 \leq j \leq k$ and

$\mathcal{E} = \{\{v_1^{i_1}, v_2^{i_2}, v_3^{i_3}, ..., v_k^{i_k}\}| v_j^{i_j} \in V_j \text{ for all } 1 \leq j \leq k \text{ and } 1 \leq i_j \leq |V_j|\}$
$k$-Partite $\sigma$-uniform hypergraph where $\sigma \geq k$
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- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, \ldots, V_k$
- $E$ is an edge if $|E| = \sigma$ and $|E \cap V_i| < \sigma$ for all $1 \leq i \leq k$. 

Jirimutu and Wang
**$k$-Partite $\sigma$-uniform hypergraph where $\sigma \geq k$**

- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, \ldots, V_k$
- $E$ is an edge if $|E| = \sigma$ and $|E \cap V_i| < \sigma$ for all $1 \leq i \leq k$.

Complete if $\mathcal{E} = \{E : |E| = \sigma$ and $|E \cap V_i| < \sigma$ for all $1 \leq i \leq k\}$
**$k$-Partite $\sigma$-uniform hypergraph where $\sigma \geq k$**

- Vertex set $V$ partitioned into $k$ subsets $V_1, V_2, V_3, \ldots, V_k$.
- $E$ is an edge if $|E| = \sigma$ and $|E \cap V_i| < \sigma$ for all $1 \leq i \leq k$.

Complete if $E = \{E : |E| = \sigma$ and $|E \cap V_i| < \sigma$ for all $1 \leq i \leq k\}$

**3-Partite 3-uniform hypergraph**

Jirimutu and Wang
3-Partite 3-uniform hypergraph
3-Partite 3-uniform hypergraph

Kuhl and Schroeder

Jirimutu and Wang
Path and Diameter of hypergraph $\mathcal{H}$
A path $P$ from $x_1$ to $x_{s+1}$ is a vertex-edge alternative sequence $x_1, E_1, x_2, E_2, \ldots, x_s, E_s, x_{s+1}$ such that $\{x_i, x_{i+1}\} \subseteq E_i$ for all $1 \leq i \leq s$ and $x_i \neq x_j, E_i \neq E_j$ with $i \neq j$ and $s$ is called the length of the path $P$. 

Ye
Path and Diameter of hypergraph $\mathcal{H}$

- A path $P$ from $x_1$ to $x_{s+1}$ is a vertex-edge alternative sequence $x_1, E_1, x_2, E_2, ..., x_s, E_s, x_{s+1}$ such that $\{x_i, x_{i+1}\} \subseteq E_i$ for all $1 \leq i \leq s$ and $x_i \neq x_j, E_i \neq E_j$ with $i \neq j$ and $s$ is called the length of the path $P$.

- The distance of distinct vertices $x$ and $y$, denoted by $d(x,y)$, is the minimum length of all paths that connect $x$ and $y$.

- The diameter of $\mathcal{H}(V, E)$, denoted by $d(\mathcal{H})$, is defined as $d(\mathcal{H}) = \max\{d(x,y) | x, y \in V, x \neq y\}$. 

Ye
Example of path and diameter
Example of path and diameter
Example of path and diameter

Path from 1 to 6
Example of path and diameter

Path from 1 to 6

1, e, 3, f, 6
1, e, 5, f, 6

d(1,6) = 2
Example of path and diameter

Path from 1 to 6

1, e, 3, f, 6
1, e, 5, f, 6

d(1,6) = 2

Path from 4 to 6

4, g, 1, e, 3, f, 6
4, g, 1, e, 5, f, 6

d(4,6) = 3
Example of path and diameter

Path from 1 to 6:
- 1, e, 3, f, 6
- 1, e, 5, f, 6

Path from 4 to 6:
- 4, g, 1, e, 3, f, 6
- 4, g, 1, e, 5, f, 6

Consider all $d(x, y)$:
- $d(1,6) = 2$
- $d(4,6) = 3$
Example of path and diameter

Path from 1 to 6

- Path 1: 1, e, 3, f, 6
- Path 2: 1, e, 5, f, 6
- $d(1, 6) = 2$

Path from 4 to 6

- Path 1: 4, g, 1, e, 3, f, 6
- Path 2: 4, g, 1, e, 5, f, 6
- $d(4, 6) = 3$

$\mathcal{d}(\mathcal{H}) = 3$
Cycle of hypergraph $\mathcal{H}$
Cycle of hypergraph $\mathcal{H}$

- Let $s \geq 2$ be an integer
Let $s \geq 2$ be an integer

An $s$-cycle is an alternating sequence,

$C = x_1, E_1, x_2, E_2, \ldots, x_s, E_s$ of distinct vertices $x_1, x_2, x_3, \ldots, x_s$

and distinct edges $E_1, E_2, E_3, \ldots, E_s$ such that $x_1, x_s \in E_s$ and

$x_i, x_{i+1} \in E_i$ for all $1 \leq i \leq s - 1$ and $s$ is called the length of

cycle $C$. 
Cycle of hypergraph $\mathcal{H}$

Let $s \geq 2$ be an integer

An $s$-cycle is an alternating sequence,

$C = x_1, E_1, x_2, E_2, ..., x_s, E_s$ of distinct vertices $x_1, x_2, x_3, ..., x_s$ and distinct edges $E_1, E_2, E_3, ..., E_s$ such that $x_1, x_s \in E_s$ and $x_i, x_{i+1} \in E_i$ for all $1 \leq i \leq s - 1$ and $s$ is called the length of cycle $C$.

If hypergraph has no cycle, such hypergraph has 0-cycle or a cycle of length 0.
Example of cycle
Example of cycle
Example of cycle

$C_1 = 5, e, 3, f$
Example of cycle

$C_1 = 5, e, 3, f$

A cycle of length 2
Example of cycle

\[ C_1 = 5, e, 3, f \]

A cycle of length 2

\[ C_2 = 4, h, 6, f, 3, e, 1, g \]
Example of cycle

\[ C_1 = 5, e, 3, f \]

A cycle of length 2

\[ C_2 = 4, h, 6, f, 3, e, 1, g \]

A cycle of length 4
$k$-Zero-divisor hypergraphs of a commutative ring $R$
\[ V = Z(R, k), \text{ set of all } k\text{-Zero-Divisors} \]
**k-Zero-divisor hypergraphs of a commutative ring** \( R \)

- \( V = Z(R, k) \), set of all \( k \)-Zero-Divisors
- \( E = \{a_1, a_2, a_3, ..., a_k\} \in \mathcal{E} \)

- \( a_1 a_2 a_3 \cdots a_k = 0 \)
- the products of elements of any \( k - 1 \) subsets of \( \{a_1, a_2, a_3, ..., a_k\} \) are nonzero

Chelvam et. al.
**k-Zero-divisor hypergraphs of a commutative ring** $R$

- $V = Z(R, k)$, set of all $k$-Zero-Divisors
- $E = \{a_1, a_2, a_3, ..., a_k\} \in \mathcal{E}$

- $a_1a_2a_3 \cdots a_k = 0$
- the products of elements of any $k-1$ subsets of $\{a_1, a_2, a_3, ..., a_k\}$ are nonzero

- We see that $k$-Zero-Divisor Hypergraphs is $k$-uniform hypergraph

Chelvam et. al.
Example of 3-zero-divisor hypergraphs of $\mathbb{Z}_{30}$
Example of 3-zero-divisor hypergraphs of $\mathbb{Z}_{30}$

$$Z(\mathbb{Z}_{30}, 3) = \{2, \bar{3}, 4, \bar{5}, \bar{8}, \bar{9}, 14, 16, 21, 22, 25, 26, 27, 28\}$$
Example of 3-zero-divisor hypergraphs of $\mathbb{Z}_{30}$

$Z(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$

$e_1 = \{2, 3, 5\}$  
$e_2 = \{2, 9, 25\}$  
$e_3 = \{2, 3, 25\}$  
$e_4 = \{2, 9, 5\}$
Complete $k$-zero-divisor hypergraph
Complete $k$-zero-divisor hypergraph

Complete $k$-partite $k$-zero-divisor hypergraph
Complete $k$-zero-divisor hypergraph

Complete $k$-partite $k$-zero-divisor hypergraph

$k$-partite $\sigma$-zero-divisor hypergraph
Construct complete $k$-zero-divisor hypergraph.

Construct complete $k$-partite $k$-zero-divisor hypergraph.

Construct $k$-partite $\sigma$-zero-divisor hypergraph.

How to construct...
Complete $k$-zero-divisor hypergraph

Complete $k$-partite $k$-zero-divisor hypergraph

$k$-partite $\sigma$-zero-divisor hypergraph

How to construct

Find diameter and minimum length of all cycles
Principal ideal domain (PID)

There exist at least $k$ prime elements $p_1, p_2, p_3, \ldots, p_k$
Ring $R$

- Principal ideal domain (PID)
- There exist at least $k$ prime elements $p_1, p_2, p_3, \ldots, p_k$

Finiteness of a ring $R/I$

Commutative Ring $R/I$
Finiteness of a ring $R$

- Principal ideal domain (PID)
- There exist at least $k$ prime elements $p_1, p_2, p_3, \ldots, p_k$

Example: $\mathbb{Z}_n \cong \mathbb{Z}/n\mathbb{Z}$

Commutative Ring $R/I$
Finiteness of a ring $R$

- Principal ideal domain (PID)
- There exist at least $k$ prime elements $p_1, p_2, p_3, \ldots, p_k$

Objective

- Find an appropriate ideal so that constructed $k$-zero-divisor hypergraph has the desired properties

Example: $\mathbb{Z}_n \cong \mathbb{Z}/n\mathbb{Z}$
Complete $k$-zero-divisor hypergraph of ring $\mathcal{R}$
Complete $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\frac{R}{Rp^k}$

Appropriate ideal: $Rp^k$
Complete $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\mathbb{R}/\mathbb{R}p^k$

Appropriate ideal: $\mathbb{R}p^k$
Complete $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\mathbb{R}/Rp^k$

Appropriate ideal: $Rp^k$

Conditions: $|\mathbb{R}p/Rp^k - \mathbb{R}p^2/Rp^k| \geq k$
Complete $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\frac{R}{Rp^k}$

Appropriate ideal: $Rp^k$

Conditions: $\left| \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k} \right| \geq k$

Vertex set: $Z\left(\frac{R}{Rp^k}, k\right) = \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k}$
Complete $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\frac{R}{Rp^k}$

Appropriate ideal: $Rp^k$

Conditions: $|\frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k}| \geq k$

Vertex set: $Z\left(\frac{R}{Rp^k}, k\right) = \frac{Rp}{Rp^k} - \frac{Rp^2}{Rp^k}$

Complete $k$-zero-divisor hypergraph
Example of complete $3$-zero-divisor hypergraph of ring $R$
Example of complete $3$-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$
Example of complete $3$-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$

A vertex set $\mathcal{Z}(\mathbb{Z}_{27}, 3) = \{3, 6, 12, 15, 21, 24\}$
Example of complete $3$-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{27} \cong \mathbb{Z}/27\mathbb{Z} \cong \mathbb{Z}/3^3\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{27}, 3) = \{3, 6, 12, 15, 21, 24\}$
Diameter of complete $k$-zero-divisor hypergraph of ring $R$
Diameter of complete \( k \)-zero-divisor hypergraph of ring \( R \)

- Diameter is 1 same as a complete graph
Diameter of complete $k$-zero-divisor hypergraph of ring $R$

- Diameter is 1 same as a complete graph
The minimum length of all cycles
The minimum length of all cycles

0 if \( |Z \left( R_{/Rp^k}, k \right)| = k \)
The minimum length of all cycles

- 0 if \( |Z\left( R_{R^p k}^{-1}, k \right)| = k \)

Only one edge
The minimum length of all cycles

0 if $|Z \left( R_{R^k P^k}, k \right)| = k$

Only one edge

2 if $k \geq 3$ and $|Z \left( R_{R^k P^k}, k \right)| \geq k + 1$
The minimum length of all cycles

➢ 0 if \( |Z\left(\frac{R}{R_p^k}, k\right)| = k \)

➢ 2 if \( k \geq 3 \) and \( |Z\left(\frac{R}{R_p^k}, k\right)| \geq k + 1 \)

\[
c = 1, e, 2, f
\]
The minimum length of all cycles

- 0 if \( |Z\left( \frac{R}{Rp}, k \right)| = k \)
- 2 if \( k \geq 3 \) and \( |Z\left( \frac{R}{Rp}, k \right)| \geq k + 1 \)
- 3 if \( k = 2 \) and \( |Z\left( \frac{R}{Rp^2}, 2 \right)| \geq 3 \)

Only one edge

\( c = 1, e, 2, f \)
The minimum length of all cycles

- 0 if $|Z(R_{/Rp^k}, k)| = k$ (Only one edge)
- 2 if $k \geq 3$ and $|Z(R_{/Rp^k}, k)| \geq k + 1$
- 3 if $k = 2$ and $|Z(R_{/Rp^2}, 2)| \geq 3$ (Same idea as complete graph)
## Conclusion table

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<th>Vertex Set</th>
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<th>Minimum length of all cycles</th>
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<td>Complete $k$-zero-divisor hypergraph</td>
<td>$Rp^k$</td>
<td>$Z\left(\frac{R}{Rp^k}, k\right) = \frac{Rp}{Rp^k} - \frac{R^2}{Rp^k}$</td>
<td>1</td>
<td>0, 2, or 3</td>
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Complete $k$-partite $k$-zero-divisor hypergraph of ring $R$
Complete $k$-partite $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\frac{R}{Rp_1p_2p_3\cdots p_k}$

Appropriate ideal: $Rp_1p_2p_3\cdots p_k$

Condition: $R$ has at least $k$ prime elements
Complete $k$-partite $k$-zero-divisor hypergraph of ring $R$

Commutative ring: $\frac{R}{R_{p_1p_2p_3 \cdots p_k}}$

Appropriate ideal: $R_{p_1p_2p_3 \cdots p_k}$

Condition: $R$ has at least $k$ prime elements
Complete $k$-partite $k$-zero-divisor hypergraph of ring $R$.

Commutative ring: $R / R_{p_1p_2p_3\cdots p_k}$

Appropriate ideal: $R_{p_1p_2p_3\cdots p_k}$

Condition: $R$ has at least $k$ prime elements

Let $\gamma = p_1p_2p_3\cdots p_k$
Complete \( k \)-partite \( k \)-zero-divisor hypergraph of ring \( R \)

Commutative ring: \( \frac{R}{Rp_1p_2p_3\ldots p_k} \)

Appropriate ideal: \( Rp_1p_2p_3\ldots p_k \)

Condition: \( R \) has at least \( k \) prime elements

Let \( \gamma = p_1p_2p_3\ldots p_k \)

Each partite set \( V_i: \frac{Rp_i}{R\gamma} - \bigcup_{j \neq i} \frac{Rp_j}{R\gamma} \)

\[ \bigcup_{i=1}^{k} V_i = Z(\frac{R}{R\gamma}, k) \]
Complete \( k \)-partite \( k \)-zero-divisor hypergraph of ring \( R \)

Commutative ring: \( \frac{R}{R_{p_1 p_2 p_3 \ldots p_k}} \)

Appropriate ideal: \( R_{p_1 p_2 p_3 \ldots p_k} \)

Condition: \( R \) has at least \( k \) prime elements

Let \( \gamma = p_1 p_2 p_3 \ldots p_k \)

Each partite set \( V_i: \frac{R_{p_i}}{R_{\gamma}} - \bigcup_{j \neq i} \frac{R_{p_j}}{R_{\gamma}} \)

\[
\bigcup_{i=1}^{k} V_i = Z^{R_{\gamma}}_{(R_{\gamma}, k)}
\]

Complete \( k \)-partite \( k \)-zero-divisor hypergraph
Example of complete $3$-partite $3$-zero-divisor hypergraph of ring $\mathbb{R}$
Consider \( \mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2\cdot3\cdot5)\mathbb{Z} \)

Example of complete 3-partite 3-zero-divisor hypergraph of ring \( R \)
Example of complete 3-partite 3-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$
Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $\mathcal{Z}(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$
Example of complete 3-partite 3-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2\cdot3\cdot5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$

\[ V_1 = \{2, 4, 8, 14, 16, 22, 26, 28\} \]
Example of complete 3-partite 3-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$

$V_1 = \{2, 4, 8, 14, 16, 22, 26, 28\}$

$V_2 = \{3, 9, 21, 27\}$
Example of complete 3-partite 3-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{\bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{8}, \bar{9}, \bar{14}, \bar{16}, \bar{21}, \bar{22}, \bar{25}, \bar{26}, \bar{27}, \bar{28}\}$

$V_1 = \{\bar{2}, \bar{4}, \bar{8}, \bar{14}, \bar{16}, \bar{22}, \bar{26}, \bar{28}\}$

$V_2 = \{\bar{3}, \bar{9}, \bar{21}, \bar{27}\}$

$V_3 = \{\bar{5}, \bar{25}\}$
Example of complete 3-partite 3-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{30} \cong \mathbb{Z}/30\mathbb{Z} \cong \mathbb{Z}/(2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{30}, 3) = \{2, 3, 4, 5, 8, 9, 14, 16, 21, 22, 25, 26, 27, 28\}$

$V_1 = \{2, 4, 8, 14, 16, 22, 26, 28\}$

$V_2 = \{3, 9, 21, 27\}$

$V_3 = \{5, 25\}$
Diameter of complete $k$-partite $k$-zero-divisor hypergraph of ring $R$
Diameter of complete $k$-partite $k$-zero-divisor hypergraph of ring $\mathcal{R}$

Diameter is 2
Diameter of complete $k$-partite $k$-zero-divisor hypergraph of ring $R$

Diameter is 2
Diameter of complete $k$-partite $k$-zero-divisor hypergraph of ring $R$

Diameter is 2
The minimum length of all cycles
The minimum length of all cycles

0 if $|Z(R/R_{\gamma}, k)| = k$
The minimum length of all cycles

- 0 if $|Z(R/R_{\gamma}, k)| = k$

Only one edge
The minimum length of all cycles

- 0 if $|Z(R_{/R'\vee}, k)| = k$

- 2 if $k \geq 3$ and $|Z(R_{/R'\vee}, k)| \geq k + 1$

Only one edge
The minimum length of all cycles

\[ Z(R_{\gamma}, k) \]

- **0** if \( \left| Z(R_{\gamma}, k) \right| = k \)

- **2** if \( k \geq 3 \) and \( \left| Z(R_{\gamma}, k) \right| \geq k + 1 \)

Only one edge

\[ c = 1, e, 2, f \]
The minimum length of all cycles
The minimum length of all cycles

- 0 if $k = 2$ and $|Z(R/Y, 2)| \geq 3$ (one of partite sets has only one element)
The minimum length of all cycles

- $0$ if $k = 2$ and $|Z(R/_{RV}, 2)| \geq 3$ (one of partite sets has only one element)
The minimum length of all cycles

- 0 if $k = 2$ and $|Z^{(R/R_Y, 2)}| \geq 3$ (one of partite sets has only one element)

- 4 if $k = 2$ and $|Z^{(R/R_Y, 2)}| \geq 3$ (each partite set has more than one element)
The minimum length of all cycles

- $0$ if $k = 2$ and $|Z^{(R/R_Y, 2)}| \geq 3$ (one of partite sets has only one element)

- $4$ if $k = 2$ and $|Z^{(R/R_Y, 2)}| \geq 3$ (each partite set has more than one element)

Same idea as complete bipartite graph
Conclusion table
## Conclusion table

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$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

How to construct $k$-partite $\sigma$-zero-divisor hypergraph
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

How to construct $k$-partite $\sigma$-zero-divisor hypergraph

Construct complete $k$-partite $k$-zero-divisor hypergraph
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

How to construct $k$-partite $\sigma$-zero-divisor hypergraph

Construct complete $k$-partite $k$-zero-divisor hypergraph

Construct complete $l$-zero-divisor hypergraph
3-partite 4-zero-divisor hypergraph of ring $R$
3-partite 4-zero-divisor hypergraph of ring $R$
3-partite 4-zero-divisor hypergraph of ring $R$

complete 3-partite 3-zero-divisor hypergraph

$Rp_1p_2p_3$
3-partite 4-zero-divisor hypergraph of ring $R$

complete 3-partite 3-zero-divisor hypergraph

$Rp_1p_2p_3$

complete 2-zero-divisor hypergraph
3-partite 4-zero-divisor hypergraph of ring $R$

- Complete 3-partite 3-zero-divisor hypergraph
  - $R_{p_1p_2p_3}$
- Complete 2-zero-divisor hypergraph
  - $Rp^2$
3-partite 4-zero-divisor hypergraph of ring $R$

- Complete 3-partite 3-zero-divisor hypergraph: $R\rho_1\rho_2\rho_3$
- Complete 2-zero-divisor hypergraph: $R\rho^2$

Incorporating: $R\rho_1^2\rho_2\rho_3$
3-partite 4-zero-divisor hypergraph of ring $R$

complete 3-partite 3-zero-divisor hypergraph

$Rp_1p_2p_3$

complete 2-zero-divisor hypergraph

$Rp^2$

$Rp^2_1p_2p_3$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$
3-partite 4-zero-divisor hypergraph of ring $R$

- Complete 3-partite 3-zero-divisor hypergraph
  - $R \ell_1 \ell_2 \ell_3$

- Complete 2-zero-divisor hypergraph
  - $R\ell^2$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z} / (2^2 \cdot 3 \cdot 5)\mathbb{Z}$

- $\bar{2}$
  - 14
- $\bar{3}$
  - 9
- $\bar{5}$
  - 35
3-partite 4-zero-divisor hypergraph of ring $R$

complete 3-partite 3-zero-divisor hypergraph

$Rp_1p_2p_3$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$
3-partite 4-zero-divisor hypergraph of ring $R$

Complete 3-partite 3-zero-divisor hypergraph

$R p_1 p_2 p_3$

Complete 2-zero-divisor hypergraph

$R p^2$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$
3-partite 4-zero-divisor hypergraph of ring $R$

complete 3-partite 3-zero-divisor hypergraph

$R_{p_1p_2p_3}$

complete 2-zero-divisor hypergraph

$R_{p_2}$

$R_{p_1^2p_2p_3}$

Ring $\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

Commutative ring: $\frac{R}{Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k}}$

Appropriate ideal: $Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k}$

Condition: $R$ has at least $k$ prime elements
A \( k \)-partite \( \sigma \)-zero-divisor hypergraph of ring \( R \) where \( \sigma \geq k \)

\[
\sigma = \sum_{m=1}^{k} \alpha_m
\]

Commutative ring: \( R / Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k} \)

Appropriate ideal: \( Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k} \)

Condition: \( R \) has at least \( k \) prime elements
\( k \)-partite \( \sigma \)-zero-divisor hypergraph of ring \( R \) where \( \sigma \geq k \)

\[
\sigma = \sum_{m=1}^{k} \alpha_m
\]

Commutative ring: \( \frac{R}{Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k}} \)

Appropriate ideal: \( Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k} \)

Condition: \( R \) has at least \( k \) prime elements

Let \( \pi = p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}...p_k^{\alpha_k} \)
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

\[ \sigma = \sum_{m=1}^{k} \alpha_m \]

Commutative ring: $\frac{R}{Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3} \cdots p_k^{\alpha_k}}$

Appropriate ideal: $Rp_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3} \cdots p_k^{\alpha_k}$

Condition: $R$ has at least $k$ prime elements

Let $\pi = p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3} \cdots p_k^{\alpha_k}$

Each partite set $V_i$:
- $\frac{Rp_i}{R\pi} \cup \bigcup_{j \neq i} \frac{Rp_j}{R\pi}$ if $\alpha_i = 1$
- $\frac{Rp_i}{R\pi} - \left( \frac{Rp_i^2}{R\pi} \cup \bigcup_{j \neq i} \frac{Rp_j}{R\pi} \right)$ if $\alpha_i \geq 2$

\[ \bigcup_{i=1}^{k} V_i = Z\left(\frac{R}{R\pi}, \sigma\right) \]
$k$-partite $\sigma$-zero-divisor hypergraph of ring $R$ where $\sigma \geq k$

$\sigma = \sum_{m=1}^{k} \alpha_m$

Commutative ring: $R / \bigcap_{1 \leq j \leq k} p_i^{\alpha_j}$

Appropriate ideal: $R \bigcap_{1 \leq j \leq k} p_i^{\alpha_j}$

Condition: $R$ has at least $k$ prime elements

Let $\pi = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \cdots p_k^{\alpha_k}$

Each partite set $V_i$:

- $R_{p_i} / R_{\pi} - \bigcup_{j \neq i} R_{p_j} / R_{\pi}$ if $\alpha_i = 1$
- $R_{p_i} / R_{\pi} - \left( R_{p_i}^2 / R_{\pi} \cup \bigcup_{j \neq i} R_{p_j} / R_{\pi} \right)$ if $\alpha_i \geq 2$

$k \sum_{i=1}^{k} V_i = Z(\frac{R}{R_{\pi}}, \sigma)$

$k$-partite $\sigma$-zero-divisor hypergraph
Example of 3-partite 4-zero-divisor hypergraph of ring $R$
Example of a 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$
Example of 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set $Z(\mathbb{Z}_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$
Example of 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$Z(\mathbb{Z}_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$
Example of **3**-partite **4**-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$V(Z_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$

$V_1 = \{2, 14, 22, 26, 34, 38, 46, 58\}$
Example of 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$Z(\mathbb{Z}_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$

$V_1 = \{2, 14, 22, 26, 34, 38, 46, 58\}$

$V_2 = \{3, 9, 21, 27, 33, 39, 51, 57\}$
Example of 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$Z(\mathbb{Z}_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$

$V_1 = \{2, 14, 22, 26, 34, 38, 46, 58\}$

$V_2 = \{3, 9, 21, 27, 33, 39, 51, 57\}$

$V_3 = \{5, 25, 35, 55\}$
Example of 3-partite 4-zero-divisor hypergraph of ring $R$

Consider $\mathbb{Z}_{60} \cong \mathbb{Z}/60\mathbb{Z} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}$

A vertex set

$Z(\mathbb{Z}_{60}, 4) = \{2, 3, 5, 9, 14, 21, 22, 25, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58\}$

$V_1 = \{2, 14, 22, 26, 34, 38, 46, 58\}$

$V_2 = \{3, 9, 21, 27, 33, 39, 51, 57\}$

$V_3 = \{5, 25, 35, 55\}$
Diameter of $k$-partite $\sigma$-zero-divisor hypergraph of ring $R$
Diameter of $k$-partite $\sigma$-zero-divisor hypergraph of ring $R$.

Diameter is 2
Diameter of $k$-partite $\sigma$-zero-divisor hypergraph of ring $R$

Diameter is 2

\[ \mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z} \]
Diameter of $k$-partite $\sigma$-zero-divisor hypergraph of ring $R$

Diameter is 2

\[ \mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z} \]
Diameter of \( k \)-partite \( \sigma \)-zero-divisor hypergraph of ring \( R \)

- Diameter is 2

\[
\mathbb{Z}_{60} \cong \mathbb{Z}/(2^2 \cdot 3 \cdot 5)\mathbb{Z}
\]
The minimum length of all cycles
The minimum length of all cycles

\[ 0 \text{ if } |Z(R/R\pi, \sigma)| = \sigma \]
The minimum length of all cycles

- $0$ if $|Z(R/R\pi, \sigma)| = \sigma$

Only one edge
The minimum length of all cycles

- 0 if $|Z(R/R\pi, \sigma)| = \sigma$
- Only one edge
- 2 if $k \geq 3$ and $|Z(R/R\pi, \sigma)| \geq \sigma + 1$
The minimum length of all cycles

- $0$ if $|Z(R/R\pi, \sigma)| = \sigma$

- $2$ if $k \geq 3$ and $|Z(R/R\pi, \sigma)| \geq \sigma + 1$

Only one edge

$c = 1, e, 2, f$
The minimum length of all cycles
The minimum length of all cycles

0 if \( k = 2 \) and \( |Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1 \)
and \( |V_1| = 1 \) and \( \alpha_2 = 1 \)
The minimum length of all cycles

- 0 if $k = 2$ and $|Z(R^{R/\pi}, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$
- and $|V_1| = 1$ and $\alpha_2 = 1$
The minimum length of all cycles

- $0$ if $k = 2$ and $|Z^{(R/R\pi, \alpha_1 + \alpha_2)}| \geq \alpha_1 + \alpha_2 + 1$
  and $|V_1| = 1$ and $\alpha_2 = 1$

- $2$ if $k = 2$ and $|Z^{(R/R\pi, \alpha_1 + \alpha_2)}| \geq \alpha_1 + \alpha_2 + 1$
  and $|V_1| = 1$ and $\alpha_2 \geq 2$
The minimum length of all cycles

- \(0\) if \(k = 2\) and \(|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1\) and \(|V_1| = 1\) and \(\alpha_2 = 1\)

- \(2\) if \(k = 2\) and \(|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1\) and \(|V_1| = 1\) and \(\alpha_2 \geq 2\)

\(c = 1, e, 2, f\)
The minimum length of all cycles
The minimum length of all cycles

- $2$ if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$
The minimum length of all cycles

2 if $k = 2$ and $|Z(R/R\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$
The minimum length of all cycles

2 if $k = 2$ and $|Z^{(R/_{R\pi}}, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$

4 if $k = 2$ and $|Z^{(R/_{R\pi}}, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ with $\alpha_i = 1$ for all $1 \leq i \leq 2$
The minimum length of all cycles

- If $k = 2$ and $|Z(R/R^\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ for all $1 \leq i \leq 2$ and there exists $1 \leq i \leq 2$ such that $\alpha_i \geq 2$

- If $k = 2$ and $|Z(R/R^\pi, \alpha_1 + \alpha_2)| \geq \alpha_1 + \alpha_2 + 1$ and $|V_i| \geq 2$ with $\alpha_i = 1$ for all $1 \leq i \leq 2$

Same idea as complete bipartite graph
Conclusion table
## Conclusion table

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<td>$(\sigma = \sum_{m=1}^{k} \alpha_m, \pi = p_1^{\alpha_1} p_2^{\alpha_2} ... p_k^{\alpha_k})$</td>
<td>$V_i = \frac{R \alpha_i}{R \pi} - \bigcup_{j \neq i} \frac{R \alpha_j}{R \pi}$ if $\alpha_i = 1$</td>
<td>$2$</td>
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$V_i = \frac{R \alpha_i}{R \pi} - \left( \frac{R \alpha_i^2}{R \pi} \bigcup \bigcup_{j \neq i} \frac{R \alpha_j}{R \pi} \right)$ if $\alpha_i \geq 2$
Recent Works
Games played on hypergraphs
Games played on hypergraphs

Cop and Robber game

Played on Graphs
Games played on hypergraphs

Cop and Robber game

Played on Graphs

Change to

Played on Hypergraphs
Games played on hypergraphs

Cop and Robber game

Played on Hypergraphs
Cop and Robber game played on graphs
Cop and Robber game played on graphs

Two players

Nowakowski and Winkler
Cop and Robber game played on graphs

Two players

Cop

Nowakowski and Winkler
Cop and Robber game played on graphs

Two players

Cop

Robber

Nowakowski and Winkler
Cop and Robber game played on graphs

Two players

Cop

Robber

Rules of the game played on graphs

Nowakowski and Winkler
Cop and Robber game played on graphs

Two players

Rules of the game played on graphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

Nowakowski and Winkler
Cop and Robber game played on graphs

Two players

Rules of the game played on graphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

2. In each round, the cop and the robber take alternatively moving from their present vertex to other vertices along edges or staying put.

Nowakowski and Winkler
Cop and Robber game played on graphs
Cop and Robber game played on graphs

How to finish the game
Copy and Robber game played on graphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves
Cop and Robber game played on graphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves
Cop and Robber game played on graphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves

Robber wins if robber can run away
**Cop and Robber** game played on graphs

**How to finish the game**

**Cop wins** if cop can catch robber by occupying the same vertex as the robber after finite number of moves

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Cop and Robber game played on graphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves

Robber wins if robber can run away
Example of cop wins and robber wins
Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of *cop* wins and *robber* wins

Given two graphs, which one that *cop* wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
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Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?

COP WINS

COP WINS
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?

COP WINS
Example of **cop** wins and **robber** wins

Given two graphs, which one that **cop** wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?
Example of cop wins and robber wins

Given two graphs, which one that cop wins?

The graph which cop wins is called a cop-win graph; otherwise, a robber-win graph.
Cop and Robber game played on hypergraphs
Cop and Robber game played on hypergraphs

Two players
Cop and Robber game played on hypergraphs

Two players

Rules of the game played on hypergraphs
Cop and Robber game played on hypergraphs

Two players

Rules of the game played on hypergraphs

1. The cop choose a beginning vertex and then the robber choose a beginning vertex
**Cop and Robber game played on hypergraphs**

Two players

**Rules of the game played on hypergraphs**

1. The cop choose a beginning vertex and then the robber choose a beginning vertex

2. In each round, they take alternatively moving from their present vertex $x$ to any vertex $y$ belonging to the same hyperedge as vertex $x$ or staying put.
Cop and Robber game played on hypergraphs
Cop and Robber game played on hypergraphs

How to finish the game
Cop and Robber game played on hypergraphs

How to finish the game

Same as playing on graphs
Cop and Robber game played on hypergraphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves. The hypergraph which cop wins is called a cop-win hypergraph

Same as playing on graphs
Cop and Robber game played on hypergraphs

How to finish the game

Cop wins if cop can catch robber by occupying the same vertex as the robber after finite number of moves. The hypergraph which cop wins is called a cop-win hypergraph.

Robber wins if robber can run away. The hypergraph which robber wins is called a robber-win hypergraph.

Same as playing on graphs
Cop and Robber game played on hypergraphs
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph

Baird
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

A path is a cop-win hypergraph
Cop and Robber game played on hypergraphs

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A cycle of length exceed 4 is a robber-win hypergraph
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Characterize cop-win hypergraphs
Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Generalize some results on graphs to hypergraphs
Cop and Robber game played on hypergraphs

- Characterize cop-win hypergraphs
- Generalize some results on graphs to hypergraphs
- Find the number of cops to catch robber when $\mathcal{H}$ is a robber-win hypergraph
Cop and Robber game played on hypergraphs

Characterize cop-win hypergraphs

Generalize some results on graphs to hypergraphs

Find the number of cops to catch robber when $\mathcal{H}$ is a robber-win hypergraph

Determine the complexity of cop and robber game played on hypergraphs
Catch me! If you can.