Stability of Rock Berm under Wave and Current Loading

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ABSTRACT

Rock berms are used for many different offshore applications. These applications include use of rock berm for protection of offshore pipelines from anchors and trawlers; for mitigating upheaval buckling of pipelines; for mitigating lateral buckling of pipelines; for scour protection. In order for the rock berm to perform its function, it is vital that the rock berm is stable under the design wave and current conditions. Thus it is vital that rock berm stability is assessed correctly. This paper presents an overview of fundamentals of rock berm stability assessment under wave and current conditions. Results presented in this paper will benefit the pipeline engineers involved in rock berm design for Front End Engineering Designs (FEED) and detailed design.

KEY WORDS: Rock berm; seabed stability; Chézy coefficient; shield parameter; threshold of motion; wave and current loading

INTRODUCTION

There are thousands of kilometers of offshore pipelines around the world, forming a network for the extraction and transportation of the oil and gas products. Rock berms are used for many different offshore applications; used for protection pipelines from anchors and trawlers; for mitigating upheaval buckling of pipelines; for mitigating lateral buckling of pipelines; for scour protection.

The rock berm stability assessment determines the minimum particle size of rock that would be stable under a given berm configuration and under design wave and current conditions. The fundamentals for the stability of rock berm calculation come from “threshold of motion of sediments”. The static stability of the seabed particles/sediments is determined by threshold of motion of the particle and the shear stresses induced on the particle by the wave and current actions. For very slow flows, the rocks/grains remain immobile. As the flow rate increases, the shear stresses applied on the particles increase and reach a threshold value at which few rocks/grains begin to move. This shear stress is known as the threshold bed shear stress. The corresponding values of current speed or the wave height is known was the threshold current speed or the threshold wave height.

For stable seabed, D_{50} of the seabed particles needs to be greater than the D_{50} corresponding to the threshold of motion.

This paper reviews the fundamentals of threshold of motion and presents the importance of Shields parameter and Chézy coefficient in rock beam stability assessment. Parametric study of seabed mobility (or rock berm stability) was undertaken for varying wave and current conditions. Results of the study are presented in graphical form to simplify the rock berm stability assessment and determine the stable rock sizing for a given rock berm geometry.

SEABED MOBILITY

The seabed mobility depends on the prevailing flow conditions. The flow conditions can be classified as:
- Current alone dominant (sites deeper than about 40 m)
- Waves alone dominant (near-shore zone in depths less than 5 m)
- Combined waves and currents (sites in depths between 5 m and 40 m)

Seabed features and substrates are influenced by the prevailing flow conditions and hence can change spatially and in time. However, the seabed sediments on the continental shelf are in long-term equilibrium (at least decadal) with the prevailing hydraulic climate. Gravel substrates correspond to those areas experiencing the strongest currents, whilst muddy sediments correspond to weak currents and areas of sand correspond to intermediate surface current strengths of 0.3 to 0.8 m/s.

ROCK BERM STABILITY METHODOLOGY

Over the years, there has been lot of research publications on threshold of motion and rock berm stability calculations. This has led to many users referring to such publications and using the equations proposed in the literature without fully understanding the overall limitations and boundaries of those equations. Different symbols used in literature for the same parameter also leads to unnecessary confusion among practicing engineers. Thus symbols and equations in this paper follow
the original reference and in some occasions the same equation is provided twice with different symbols to avoid any confusion. This paper aims to provide the two main fundamental methodologies that can be used for the seabed mobility assessment. These are;

1. Methodology 1- based on Shields parameter (Soulsby, 1997)

**METHODODOLOGY 1**

In this methodology, the stability of the rock-berm is determined by threshold of motion shear stress of the rock particle and the shear stressed induced on the rock particle by the wave and current action. For stable rock-berm, the D\(_{50}\) of the rock-berm needs to be greater than the D\(_{50}\) corresponding to the threshold of motion.

**Threshold Bed Shear stress**

The threshold bed shear stress is commonly expressed in non-dimensional form as critical Shields parameter.

\[
\theta = \frac{\tau_{cr}}{g \rho (s-1)d}
\]

Where

\[\tau_{cr} = \text{threshold bed shear stress (N/m}^2)\]
\[\rho = \text{density of water (kg/m}^3)\]
\[s = \text{ratio of density of seabed particle to density of water}\]
\[d = \text{particle diameter (taken as } D_{50}\text{)}\]
\[g = \text{acceleration due to gravity (9.81 m/s}^2)\]
\[D_{50} = \text{also known as median diameter or medium value of particle diameter, it is particle diameter value at which cumulative distribution percentage reaches 50\%}.\]

The critical Shields parameter for sands on initially flat horizontal bed can be calculated as below.

\[
\theta = \frac{0.30}{1 + 1.3D_s} + 0.055[1 - \exp(-0.02D_s)]
\]

where

\[D_s = \left[\frac{g(s-1)}{v^2}\right]^{1/3} d\]
\[V = \text{kinematic viscosity of water } 1.5\times10^-6 \text{ m}^2/\text{s}\]

The research in seabed movements, Panagiotopoulos et al.(1997) has shown that the cohesive material generally increases the erosion threshold of sandy deposits. The critical bed shear stresses required to displace fine-grained sands increased by up to 90% with the addition of ~10% clay. For higher clay contents the threshold shear stress can increase by 2-3 folds, hence seabed with silty clayey sands are less susceptible than pure sands to wave and current induced movements.

If the seabed mobility assessment is for mixture of mud and sand, then Mitchener et al, 1996, presented the equation below for critical bed shear stress.

\[
\tau_{cr} = 0.015(\rho_b - 1000)^{0.73}
\]

where \(\rho_b\) is the bulk density (kg/m\(^3\)) of sediments.

If the bed is sloping or stability is for a rock-berm with a slope, then gravity provides a component of the force on the grain which may increase or decrease the threshold shear stress required for mobility depending of the slope direction. The threshold bed shear stress, \(\tau_{fs}\), for a grains on a bed sloping at an angle, \(\beta\), to the horizontal in a flow making an angle, \(\psi\), to upslope direction can be related to the threshold critical shear stress value for the same grain on a flatbed by the following equation.

\[
\frac{\tau_{fs}}{\tau_{cr}} = \cos \psi \sin \beta + \left[\cos^2 \beta \tan^2 \phi - \sin^2 \psi \sin^2 \beta\right]^{1/2}
\]

where

\[\psi = \text{incident angle to slope}\]
\[\beta = \text{angle of repose (taken as 32 deg in this paper)}\]

\[\phi = \text{angle of repose (taken as 32 deg in this paper)}\]

**Bed shear stress**

The bed shear stress acting on the bed is made up from contributions due to

- Skin friction
- Form drag
- Sediment-transport (caused by momentum transfer)

It is important to note that only skin friction contribution acts directly on the sediment grains and it is therefore this contribution which is used to calculate the threshold of motion, bedload transport and pick-up rate for grains in suspension. On the other hand, it is the total bed shear stress that corresponds to the overall resistance of the flow and determined the turbulence intensities which influence the diffusion of the suspended sediment to higher levels in water column. When the bed is flat and sediment transport is not too intense, then the total bed shear stress is equal to the skin-friction contribution.

1. **Bed shear stress due to current**

Note that the shear stress due to current (\(\tau_c\)) can be calculated using different methods (There can be 50% difference between the largest and lowest calculated values of \(\tau_c\) estimates). In this paper, the shear stress due to current, \(\tau_c\), is calculated using friction velocity, \(u^*\), as below.

\[
\tau_c = \rho u^2
\]

where

\[
u_*, = 0.12\left(\frac{d_{50}}{z}\right)^{1/7} U(z)
\]

\[z = \text{distance from seabed}\]
\[\rho = \text{density of water (kg/m}^3)\]
\[d_{50} = D_{50}\text{ particle diameter (taken as } D_{50}\text{)}\]
Alternatively, drag coefficient can be used to calculate the shear stress as shown below.

\[
\tau_c = \rho C_D U^2
\]  \hspace{1cm} (6)

where

\[U = \text{depth averaged velocity}\]
\[C_D = \text{drag coefficient}, \text{(this can be taken as 0.0025 if no information is available)}\]

\[\theta = \text{bed shear stress due to current alone}\]
\[\theta = \text{shear stress (N/m²)}\]
\[\delta = \text{bed shear stress due to wave alone}\]
\[\rho = \text{apparent mass density of current, or by relating to Chezy coefficient} C\]

\[C_0 \text{ can be calculated either based on a logarithmic velocity profile in current, or by relating to Chezy coefficient} C.\]

A. Based on a logarithmic velocity profile in current, \(C_0\) can be calculated as below.

\[
C_0 = \left[\frac{0.40}{1 + \ln(z_0/h)}\right]^2
\]  \hspace{1cm} (7)

Where \(z_0 = d_{50}/12\) and \(h\) is water depth.

B. Based on Chezy coefficient \(C\), \(C_D\) can be calculated as below.

\[
C_D = \frac{g}{C^2}
\]  \hspace{1cm} (8)

where

\[C = 18 \log(1 + 12h/k_s)\]
\[k_s \text{ is hydraulic roughness} (\text{made of grain roughness and bed form roughness}); \text{for flat bed,} \ k_s \text{ can range from 2D}_{50} \text{ for fine sediments to 4D}_{50} \text{ for coarse sediments.} \ k_s \text{ is taken as 2.5} \ D_{50}\]

\[\psi = \text{Bed shear stress due to wave}\]

\[
\tau_w = \frac{1}{2} \rho f_w U_{w}^2
\]  \hspace{1cm} (9)

where

\[U_w = \text{bottom orbital velocity} \sqrt{2} U_{rms} \text{ (where} U_{rms} \text{ is obtained from JONSWAP spectrum)}\]
\[f_w = \text{wave friction factor} 1.39 (A/z_0)^{-0.52}, \text{ where}\]
\[z_0 = d_{50}/12\], and semi orbital excursion \(A = U_{rms} T_p/2\pi\).
\((T_p) \text{ is the period associated with the peak of the wave energy spectrum.}\)

3. 

\[\psi = \text{Bed shear stress due to combined current and wave}\]

The bed shear stresses beneath combined waves and currents are enhanced beyond the values which would result from a simple linear addition of the wave-alone and current alone stresses. This is because of a non-linear interaction between the wave and current boundary layers. There are more than 20 different theories and models which aim to capture this interaction. The best fit to most models and data have been captured by the following mean shear stress, \(\tau_m\).

\[
\tau_m = \tau_c \left[1 + 1.2 \left(\frac{\tau_w}{\tau_c + \tau_w}\right)^{3.2}\right]
\]  \hspace{1cm} (10)

Where \(\tau_c\) = bed shear stress due to current alone
\(\tau_w\) = bed shear stress due to wave alone

The maximum bed shear stress is then obtained by,

\[
\tau_{max} = \left[(\tau_m + \tau_w \cos \theta)^2 + (\tau_w \sin \theta)^2\right]^{1/2}
\]  \hspace{1cm} (11)

Where \(\theta\) is angle between current direction and direction of wave travel. \(\tau_{max}\) is used for the calculation of threshold of motion and entrainment rate of sediments. When the wave and current induced \(\tau_{max}\) is equal to or greater than the threshold bed shear stress, \(\tau_{cr}\), the bed is mobile.

METHODOLOGY 2

The traditional design method for the hydraulic stability of rock fill is based on the incipient motion or critical shear concept. For unidirectional steady flow the initial instability of bed material particles on a horizontal, plane bed is described by the Shields criterion same as Eq (1), but in section 5.2.1.3 of CIRIA Rock Manual the equation is presented with different format with different symbols as below.

\[
\psi_{cr} = \frac{\tau}{(\rho_r - \rho_g)gD}
\]  \hspace{1cm} (12)

where \(\tau\) is the shear stress (N/m²); \(\rho_r\) is the apparent mass density of the stones (kg/m³); \(D\) is the sieve size (m) taken as D50.

Threshold Bed Shear Stress

This criterion essentially expresses the critical value of the ratio of the de-stabilising fluid forces (that tend to move the particle) to the stabilising forces acting on a particle. The forces that tend to move the bed material particle are related to the maximum shear stress exerted on the bed by the moving fluid; the stabilising forces are related to the submerged weight of the particle. When the ratio of the two forces, represented by the shear stress (Shields) parameter, \(\psi\), exceeds a critical value, \(\psi_{cr}\), movement is initiated.

\[
\psi_{cr} = 0.03 - 0.035 \text{ for point at which stones first begin to move}
\]
\[
\psi_{cr} = 0.05 - 0.055 \text{ for limited movement}
\]

1. Bed shear stress due to current

CIRIA Rock Manual calculates current-induced shear stress, \(\tau_c\) (N/m²), acting on the bed in steady flow using the following equation, based on Chézy’s roughness equation:

\[
\tau_c = \rho_u g (U/C)^2
\]  \hspace{1cm} (13)

where \(U\) is the depth-averaged current velocity (m/s) and \(C\) is the Chézy coefficient (m²/s) (18 Log (1 + 12h/k_s)). When the bed is hydraulically rough (u*k/V > 70; as for rock) the value of \(C\) depends only on the water depth, \(h\) (m), and the bed roughness, \(k_s\) (m).

For uniform sediment the range of grain roughness is given by \(k_s/D_{50} = 1\) to 2. Despite scatter, on average the best results seem to be obtained using \(k_s = D_{90} \equiv 2 \times D_{50}\) for fine sediments and \(k_s = 2\)
\[ xD_{90} \approx 4 \times D_{50} \] for coarse material, assuming no bed-form roughness. Thus, \( k_s = 4 \times D_{50} \) can be used for rocks.

2. Bed shear stress due to wave

CIRIA Rock Manual recommends the following equation between maximum shear stress under oscillatory flow, \( \tau_w \) (N/m\(^2\)), and the relevant hydraulic parameters.

\[ \tau_w = \frac{1}{2} \rho_w f_w u_0^2 \]  

(14)

where \( f_w \) is the friction factor (-) and \( u_0 \) is the peak orbital velocity near the bed (m/s\(^2\)), which may be determined, as a first approximation, by linear wave theory. \( f_w \) is determined assuming \( z_0 = k_s/30 \) using

\[ f_w = 0.237 \left( \frac{a_0}{k_s} \right)^{-0.52} \]  

(15)

Where \( a_0 \) (m) is the amplitude of horizontal orbital wave motion at the bed as defined below (T is wave period).

\[ a_0 = u_0 \frac{T}{2\pi} \]  

(16)

3. Bed shear stress due to combined current and wave

For the incipient motion of coarse material in oscillatory flow, the Shields criterion for the initiation of motion is taken as \( \Psi_{cr} = 0.03 \) and the average oscillatory shear stress is used to calculate the critical shear stress, (N/m\(^2\)), evaluated using \( \tau_c + (0.5 \times \tau_w) \). Thus based on this methodology, the particles start to be mobile when \( \tau_c + (0.5 \times \tau_w) \)

\[ 0.03(\rho_f - \rho_w)gD_{50}. \]

PARAMETRIC STUDY ON STABILITY

Based on methodology 1, parametric study was undertaken to determine the stable \( D_{50} \) under wave and current conditions in flat seabeds. \( H_s \) (significant wave height) was ranged from 1m to 12m, \( T_p \) was taken as 8s (\( T_p \) is period associated with the peak of the wave energy spectrum, \( T_z \) which is zero crossing period = 0.781 \( T_p \)). \( U_c \) (current velocity at 1m from seabed) was varied from 0.5m/s to 3m/s. Results of parametric study are presented in graphical form from Fig. 2 to Fig. 9.

This paper introduces an Amplification factor (AF). The AF is multiplied by the stable \( D_{50} \) for flat seabed (from Fig. 2 to Fig. 5) in order to obtain the stable \( D_{50} \) for rock berms with side slopes (1:4 or 1:3).

WORKED EXAMPLE

Let us assume we have to find out the stable \( D_{50} \) in water depth of 20m with \( U_c \) of 0.5 m/s, \( H_s \) of 4m and \( T_p \) of 8s. We can use Fig.2 to find the stable \( D_{50} \) it can be seen that the stable \( D_{50} \) is \( \approx 7.5 \)mm. Let us assume that we now need to find out, what should be the stable \( D_{50} \) for a rock berm with 1:3 side slope under the same wave and current conditions. We already know that the stable \( D_{50} \) flat seabed is \( \approx 7.5 \)mm. We need to use Fig. 8 to conclude that the amplification factor (on \( D_{50} \) size) is \( \approx 4.5 \).

Thus the \( D_{50} \) for 1:3 rock berm is \( \approx 33.75 \)mm (\( 7.5 \times 4.5 \)). If we use the equations, we will derive 34.5mm. Thus the use of chat is a quick way to obtain \( D_{50} \) for rock beam design.

If we use Method 2 to find out the stable \( D_{50} \) for the same scenario considered above, the resulting \( D_{50} \) for flat bed is \( \approx 7.88 \)mm, which is close to the value obtained from Method 1. Here, \( k_s \) has been considered as \( 2 \times D_{50} \) as the particle size is more appropriate to be taken as fine particle.

DISCUSSION

Stability of seabed grains is based on comparing the wave and current induced shear stress on the seabed with critical bed-shear stress. This concept is the same for both methodologies 1 & 2, but the evaluation of critical stress and the method of combining current induced shear and wave induced shear are different in the methodologies. Thus it is vital that the two methodologies are not mixed and they should be applied independently under the limitations of the methodology.

The methodology 2 – based on CIRIA Rock Manual produce lower induced shear stresses than Methodology 1 (based on Soulby, 1997). But it is important to note that this difference is offset by three facts that are key for using the CIRIA methodology correctly.

1. Induced shear stress

the CIRIA Rock Manual states, (pg 548)

“where the critical shear stress is based on the average shear stress under oscillatory flow \( \frac{1}{2} \bar{\tau}_w = \frac{1}{2} \tau_w \), the shield parameter should have a value of 0.03, to agree with the results Rance and Warren (1968)"

Whereas, the methodology 1 (Soulby, 1997) provides the shield parameter based on D* and not fixed at 0.03, thus its methodology of combining wave and current to obtain the induced shear stresses is different to that of methodology 2.

2. Limitation of Methodology 2 (CIRIA Rock Manual)

It is to be noted that the CIRIA Rock Manual equation is applicable only under the condition,

\[ \tau_c > 0.40 \cdot \tau_w \]  

(17)

If equation (16) is not satisfied (i.e in relatively strong waves combination with weak current), CIRIA Rock Manual recommends the use of Methodology 1.

3. Amplification factor in Methodology 2

CIRIA Rock Manual provides an Amplification factor \( (k_w, \text{ equation } 5.113 \text{ of the rock manual}) \) for the bed shear stress as a result of waves superimposing upon current.

\[ k_w = 1 + \frac{1}{2} f_w \left( \frac{C_k}{2g} \left[ \frac{u_0}{U} \right]^2 \right) \]  

(18)
Fig. 2. Stable $D_{50}$ under $U_c = 0.5 \text{m/s}$, $T_p = 8 \text{s}$, $H_s = 1 \text{m}$ to $12 \text{m}$.

Fig. 3. Stable $D_{50}$ under $U_c = 1 \text{m/s}$, $T_p = 8 \text{s}$, $H_s = 1 \text{m}$ to $12 \text{m}$. 
Fig. 4. Stable $D_{50}$ under $U_c = 2\, \text{m/s}$, $T_p = 8\, \text{s}$, $H_s = 1\, \text{m}$ to $12\, \text{m}$.

Fig. 5. Stable $D_{50}$ under $U_c = 3\, \text{m/s}$, $T_p = 8\, \text{s}$, $H_s = 1\, \text{m}$ to $12\, \text{m}$. 

The graphs show the relationship between water depth and stable $D_{50}$ for different conditions.
Fig. 6. Effect of $T_p$ on stable $D_{50}$ under $U_c = 1 \text{ m/s}$, $H_s = 1\text{m}$ to $3\text{m}$.

Fig. 7. Amplification on stable $D_{50}$ for rock berm of 1:4 under $U_c = 0.5\text{m/s}$, $T_p = 8\text{s}$, $H_s = 1\text{m}$ to $10\text{m}$.

Fig. 8. Amplification on stable $D_{50}$ for rock berm of 1:3 under $U_c = 0.5\text{m/s}$, $T_p = 8\text{s}$, $H_s = 1\text{m}$ to $10\text{m}$.

Fig. 9. Amplification on stable $D_{50}$ for rock berm of 1:4 & 1:3 under $U_c = 0.5\text{m/s}$, $T_p = 8\text{s}$, $H_s = 3\text{m}$.

Fig. 6 shows the effect of $T_p$ on the $D_{50}$ for given $H_s$. It can be concluded that effect of $T_p$ is only prominent for water depths below $\sim 5\text{m}$.

Fig. 7 and Fig. 8 are amplification factor (AF) to be applied to the $D_{50}$ obtained for flat seabed (from Fig. 2 to Fig. 5) for rock beams with side slope 1:4 and 1:3 respectively.

Fig. 9 shows the effect of $U_c$ on the amplification factor. As $U_c$ increases ($U_c$ is the current velocity 1m above seabed), the AF decreases for water depth below $\sim 15\text{m}$ and increased for water depths above $\sim 15\text{m}$.

CONCLUSION
This paper presented two fundamental methodologies (Methodology 1- based on Shields parameter (Soulsby, 1997) and Methodology 2- based on CIRIA Rock Manual (2007)) for stability assessment of rock beams.

Key factors to be considered in using the methodologies correctly were highlighted in this paper. Parametric study (based on Methodology 1) was undertaken for ranging water depths, wave heights and current conditions. The results were presented in graphical form such that these can be used as design charts by engineers to easily and quickly obtain the stable $D_{50}$ size for rock on flat seabed or rock berms of 1:3 or 1:4 slopes with varying water depths (0-300m).

REFERENCES